CHAPTER 01

GEOMETRY

GEOMETRY → Lines & Angle | Triangle | Quadrilateral | Polygon | Circle

LINES & ANGLE

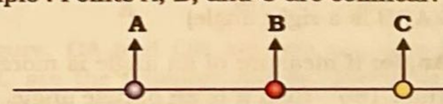
Point

A point is a circle of zero radius which determines a location. It is usually denoted by a capital letter.

Types of Points → Collinear Points | Non-Collinear Points | Intersecting Points

(a) Collinear Points: If three or more points situated on a straight line, these points are called collinear points.

Example : Points A, B, and C are collinear.



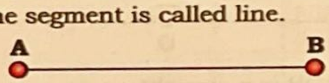
(b) Non-collinear Points: If three or more points are not situated on a straight line, these points are called non-collinear points.

(c) Intersecting Points: When two or more lines cross or meet at a common point, that point is known as intersecting point.

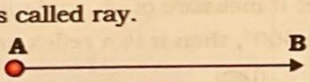
Line

A line is made up of an infinite number of points and it has only length i.e., it does not have any thickness (or width). A line is endless so, it can be extended in both directions.

Line Segment: A line segment has two end points; generally line segment is called line.



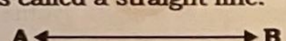
Ray: A ray extends indefinitely in one direction from any given point is called ray.



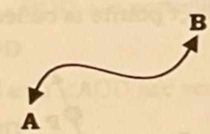
Name	Line	Line segment	Ray
Symbol	$\overleftrightarrow{AB}$	$\overline{AB}$	$\overrightarrow{AB}$
End Points	No end points	Two end points	One end points

Types of Lines

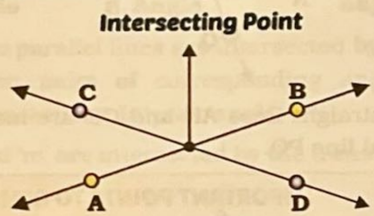
(a) Straight line: A line which does not change its direction at any point is called a straight line.



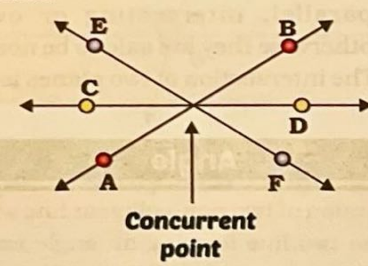
(b) Curve line: A line which changes its direction is called a curved line.



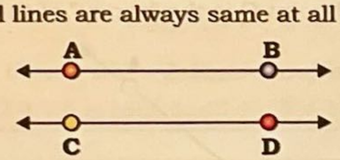
(c) Intersecting Lines: If two or more lines intersect each other, then they are called intersecting lines. In the figure AB and CD are intersecting lines. Two lines can intersect maximum at one point.



(d) Concurrent Lines : If three or more lines pass through a point, then they are called concurrent lines and the point through which these all lines pass is called point of concurrent.



(e) Parallel Lines : Two straight lines are parallel if they lie in the same plane and do not intersect even if they produced infinitely. Perpendicular distances between two parallel lines are always same at all places.

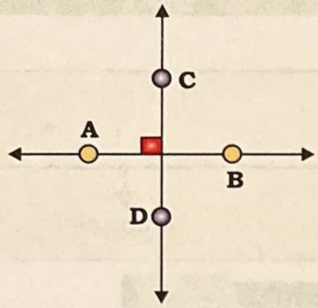


In the figure AB and CD are parallel lines.

Symbol for parallel lines is | |.

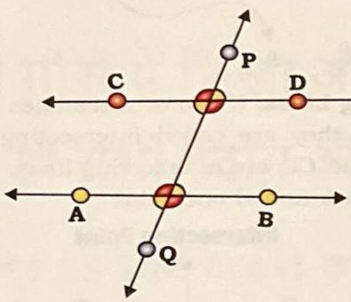
Hence, AB | | CD.

(f) Perpendicular Lines : If two lines intersect each other at right angles, then two lines are called perpendicular lines. In the following figure AB and CD are perpendicular lines.



Symbolically it is represented as AB ⊥ CD or we can also say that CD ⊥ AB.

(g) Transversal Lines : A line which intersects two or more given lines at distinct points is called a transversal of the given lines.



In figure straight lines AB and CD are intersected by a transversal line PQ.

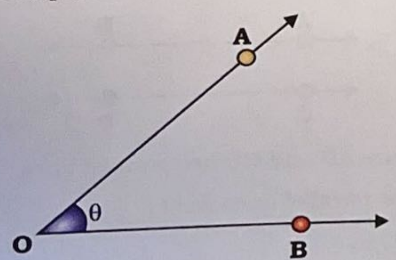
IMPORTANT POINTS TO REMEMBER

- There is one and only one line passing through two distinct points.
- Two or more lines are said to be coplanar if they lie in the same plane and can be parallel, intersecting or overlapping, otherwise they are said to be non-coplanar.
- The intersection of two planes is a line.

Angle

An angle is the union of two non-collinear line with a common initial point. The two line forming an angle are called arms of the angle and the common initial point is called the vertex of the angle.

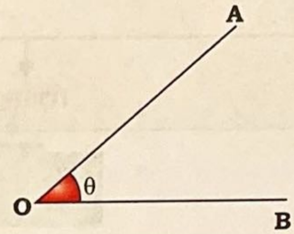
- The angle AOB is denoted by ∠AOB, is formed by line OA and OB and point O is the "vertex" of the angle.



(I) **Types of Angle**

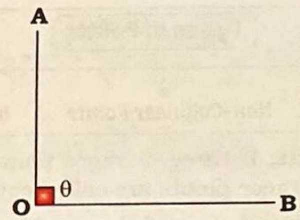
- Acute angle
- Right angle
- Obtuse angle
- Straight angle
- Reflex angle
- Complete angle

Acute Angle: If the measure of an angle is less than 90°, it is an acute angle.



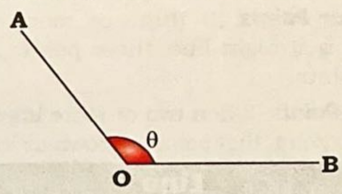
$0° < \theta < 90°$ (∠AOB is an acute angle)

Right Angle : If measure of an angle is equal to 90°, then it is a right angle.



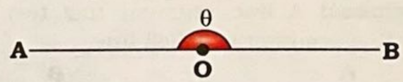
$\theta = 90°$ (∠AOB is a right angle)

Obtuse Angle: If measure of an angle is more than 90° but less than 180°, then it is an obtuse angle.



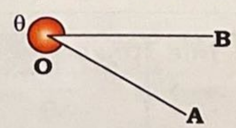
$90° < \theta < 180°$ (∠AOB is an obtuse angle)

Straight Angle: If measure of an angle is equal to 180°, then it is a straight angle.



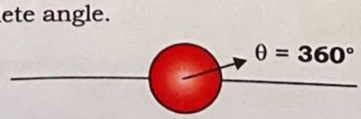
$\theta = 180°$ (∠AOB is a straight angle)

Reflex Angle: If measure of an angle is more than 180° but less than 360°, then it is a reflex angle.



$180° < \theta < 360°$ (∠AOB is a reflex angle)

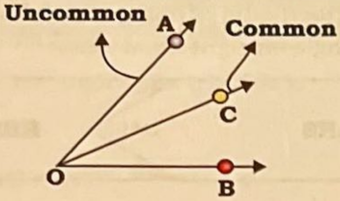
Complete angle: If measure of an angle is 360° then it is a complete angle.



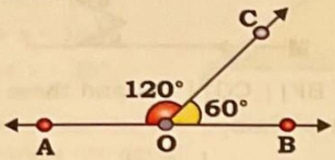
(II) Pair of Angles

- Adjacent Angle
- Linear Angle
- Complementary Angle
- Supplementary Angle

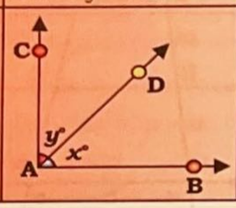
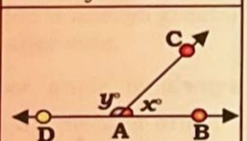
Adjacent Angles: Two angles are called adjacent angles if:
- They have the same vertex,
- They have a common arm,
- Uncommon arms are on either side of the common arm.



- In the figure, ∠AOC and ∠BOC have a common vertex O. Also, they have a common arm OC and the distinct arms OA and OB, lie on the either side of common arm OC.
- **Linear Pair of Angles:** Two adjacent angles are said to form a linear pair of angles, if their uncommon arms are two opposite line.



- In figure, OA and OB are two opposite line ∠AOC & ∠BOC are the adjacent angles. Therefore, ∠AOC and ∠BOC form a linear pair.
- If a line stand on another line, the sum of the adjacent angles so formed is 180°.

Angle	Complementary	Supplementary
Definition	If sum of two angles is equal to 90°, then the two angles are called complementary angles. • ∠BAD and ∠DAC are complementary angles, if $x^o + y^o = 90°$	If sum of two angles is equal to 180°, then the two angles are called supplementary angle. • ∠BAC and ∠DAC are supplementary angles, if $x^o + y^o = 180°$
Representation		

Ex. The measure of an angle for which the measure of the supplement is four times the measure of the complement is:

HINTS Let Angle = x.

So, Complementary angle = $90° - x$

Supplementary angle = $180° - x$

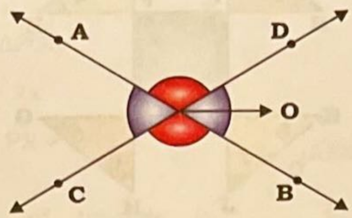
ATQ,

4 × Complementary angle = Supplementary angle

$\Rightarrow 4 \times (90° - x) = 180° - x \Rightarrow x = 60°$

(III) Vertically Opposite Angles

If arm of two angles form two pairs of opposite rays, then the two angles are called as vertically opposite angles.



- In other words, when two lines intersect, two pairs of vertically opposite angles are formed. Each pair of vertically opposite angle is equal.
- In the figure, two lines AB and CD intersect at O. We find that ∠AOC and ∠BOD are vertically opposite angles

$\Rightarrow$ ∠AOC = ∠BOD

Similarly, ∠BOC and ∠AOD are vertically opposite angles.

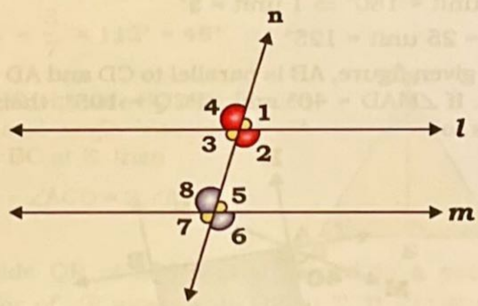
$\Rightarrow$ ∠BOC = ∠AOD

(IV) Angles made by Transversal Lines

- Corresponding Angle
- Alternate Angle
- Consecutive Interior Angle

When two parallel lines are intersected by a transversal. They form pairs of corresponding angles, Alternate angle, Consecutive Interior angles.

Lines' and 'm' are intersected by the transversal 'n'. Then



Corresponding Angle

∠1 = ∠5, ∠4 = ∠8, ∠3 = ∠7 and ∠2 = ∠6

Alternate Interior Angle

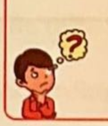
∠3 = ∠5 and ∠2 = ∠8

Alternate Exterior Angle

1 = ∠7 and ∠4 = ∠6

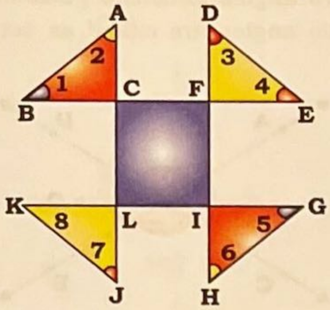
Consecutive Interior Angles

∠2 + ∠5 = 180° & ∠3 + ∠8 = 180°



Conversely, if a transversal intersects two lines in such a way that a pair of alternate interior angles is equal, then the two lines are parallel.

Ex. Angles are shown in the given figure. What is the value of $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8$?



HINTS $\angle C = 180° - (\angle 1 + \angle 2) \Rightarrow \angle F = 180° - (\angle 3 + \angle 4)$

$\Rightarrow \angle I = 180° - (\angle 5 + \angle 6) \Rightarrow \angle L = 180° - (\angle 7 + \angle 8)$

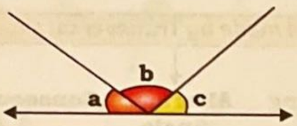
CFIL is quadrilateral.

$\therefore$ Sum of angles of CFIL is 360°.

$\Rightarrow 720° - (\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8) = 360°$

$\Rightarrow 360° = (\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8)$

Ex. In the given figure, if $\frac{b}{a} = 5$ and $\frac{c}{a} = \frac{6}{5}$ then what is the value of b.



HINTS $b : a = (5 : 1) \times 5 = 25 : 5$

$c : a = 6 : 5$

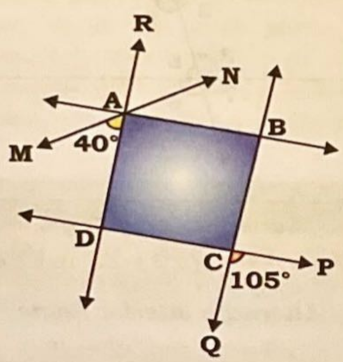
$\therefore a : b : c = 5 : 25 : 6$

$\angle a + \angle b + \angle c = 180°$ (Straight line)

$\Rightarrow 36 \text{ unit} = 180° \Rightarrow 1 \text{ unit} = 5°$

$\therefore \angle b = 25 \text{ unit} = 125°$

Ex. In the given figure, AB is parallel to CD and AD is parallel to BC. If $\angle MAD = 40°$ and $\angle PCQ = 105°$, then $\angle NAB$ is equals to?



HINTS

$\angle BCD = \angle PCQ = 105°$ (Vertically opposite angle)

$\angle CDA = 180° - 105° = 75°$

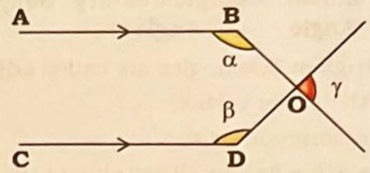
$\angle RAB = \angle CDA = 75°$ (corresponding angles)

$\angle RAN = \angle MAD = 40°$ (Vertically opposite angle)

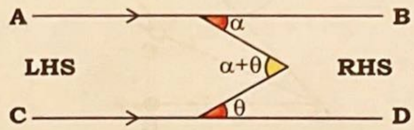
$\therefore \angle NAB = \angle RAB - \angle RAN = 75° - 40° = 35°$

Some Important Results

☞ If AB||CD then $\alpha + \beta + \gamma = 360°$



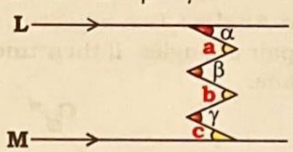
☞ If AB||CD, then sum of angle on left hand side is equal to sum of angle on right hand side.



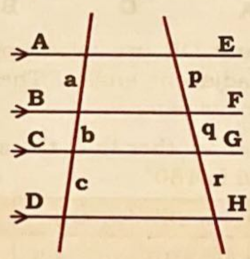
RHS = LHS

$\Rightarrow \theta + \alpha = \alpha + \theta$

☞ If L || M, $a + b + c = \alpha + \beta + \gamma$



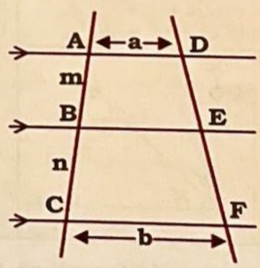
☞ When AE|| BF|| CG|| DH and these lines are cut by two transversal lines.



Then, $a : b : c = p : q : r$

$$\frac{a}{a+b+c} = \frac{p}{p+q+r}$$

☞ When AD|| BE||CF and these lines are cut by two transversal lines.

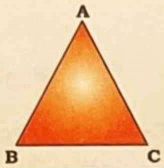


Then, $\frac{AB}{BC} = \frac{DE}{EF} = \frac{m}{n}$

$$BE = \frac{an + bm}{m + n}$$

TRIANGLE

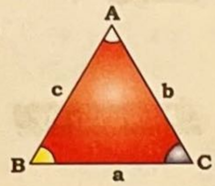
A triangle is a polygon with three sides and three angles. It is the fundamental shape in geometry formed by connecting three non-collinear points.



Fundamental Properties of Triangle

☞ Sum of all three angles of a triangle is always 180°

$\angle A + \angle B + \angle C = 180°$



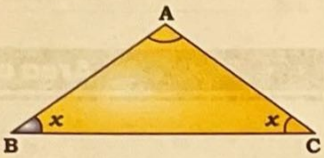
Ex. One of the angles of a triangle is 108° and the other two angles are equal. What is the measure of each of these equal angles?

HINTS We know,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 108° + x + x = 180°$

$\Rightarrow 2x = 72° \Rightarrow x = 36°$



☞ (i) Angles opposite to the equal sides of a triangle are equal.

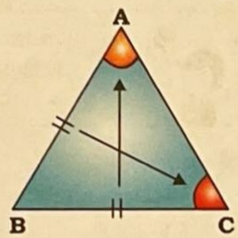
(ii) Sides opposite to the equal angles of a triangle are equal.

In ΔABC, If AB = BC, then

$\angle A = \angle C$

In ΔABC, If $\angle A = \angle C$, then

AB = BC

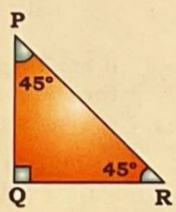


Ex. In ΔPQR, $\angle Q = 90°$, PQ = 8 cm and $\angle PRQ = 45°$. Find the length of QR.

HINTS

Using property 2(ii),

QR = PQ = 8 cm



☞ (i) The angle opposite to the greater side is always greater than the angle opposite to the smaller side.

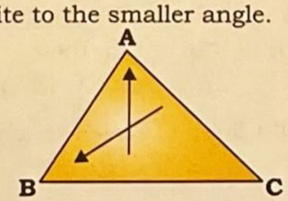
(ii) The side opposite to the greater angle is always greater than the side opposite to the smaller angle.

In ΔABC, If BC > AC, then

$\angle A > \angle B$

In ΔABC, If $\angle A > \angle C$, then

BC > AB



Ex. The ratio of the angles, $\angle P$, $\angle Q$ and $\angle R$ of a ΔPQR is 2 : 4 : 9, then which of the following is true?

(a) PQ > QR > RP

(b) PQ > RP > QR

(c) QR > RP > PQ

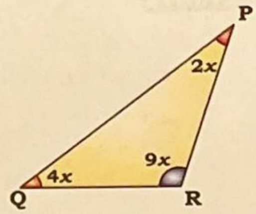
(d) PR > PQ > QR

HINTS

Here, In ΔPQR

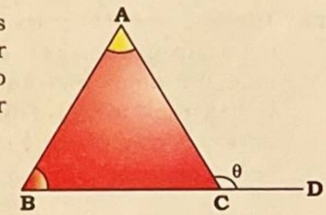
$9x > 4x > 2x$

So, PQ > PR > QR



☞ If a side of triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

$\angle ACD = \angle CAB + \angle ABC$



Ex. The side BC of ΔABC is produced to D. If $\angle ACD = 112°$ and $\angle B = \frac{3}{4}\angle A$, then find the measure of $\angle B$.

HINTS We know,

Exterior angle ($\angle ACD$)

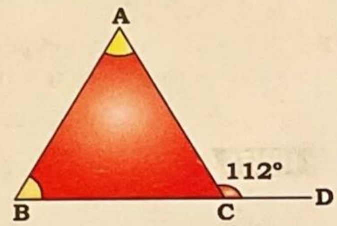
$= \angle A + \angle B$

$\because \angle B = \frac{3}{4}\angle A$

$\Rightarrow \angle A = \frac{4}{3}\angle B$ ....(i)

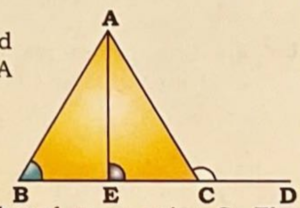
$\therefore 112° = \frac{4}{3}\angle B + \angle B$

$\Rightarrow \angle B = \frac{3}{7} \times 112° = 48°$



☞ In ΔABC, the side BC is produced to D and angle bisector of $\angle A$ meets BC at E, then

$\angle ABC + \angle ACD = 2\angle AEC$.



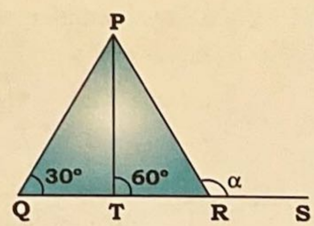
Ex. The side QR of ΔPQR is produced to a point S. The bisector of $\angle P$ meets side QR at T. If $\angle PQR = 30°$ and $\angle PTR = 60°$, find $\angle PRS$.

HINTS

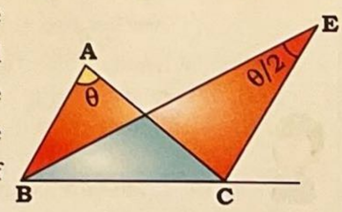
$\angle PQR + \angle PRS = 2 \times \angle PTR$

$\Rightarrow 30° + \alpha = 2 \times 60°$

$\Rightarrow \alpha = 120° - 30° = 90°$



☞ In a triangle, the angle formed between internal bisector of one base angle and external bisector of the other base angle is half of the remaining vertex angle.



According to this property, $\angle BEC = \frac{\angle A}{2}$.

Ex. In ΔABC, $\angle B = 72°$ and $\angle C = 44°$. Side BC is produced to D. Then bisectors of $\angle B$ and $\angle ACD$ meet at E. What is the measure of $\angle BEC$?

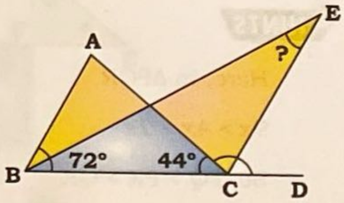
HINTS We know,

$\angle A + \angle B + \angle C = 180°$

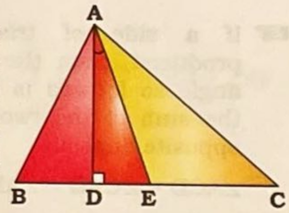
$\Rightarrow \angle A = 180° - 72° - 44°$

$\Rightarrow \angle A = 64°$

$\therefore BEC = \frac{\angle A}{2} = \frac{64°}{2} = 32°$



☞ The angle between perpendicular drawn from a vertex to opposite side and angle bisector of that vertex angle is half of difference between other two remaining vertex angles.



If $AD \perp BC$ and AE is angle bisector of $\angle A$, then

$\angle DAE = \left|\frac{\angle B - \angle C}{2}\right|$

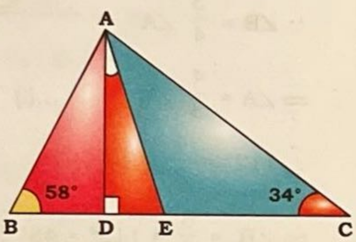
Ex. In ΔABC, AD is perpendicular to BC and AE is the bisector of $\angle BAC$. If $\angle ABC = 58°$ and $\angle ACB = 34°$, then find the measure of $\angle DAE$.

HINTS

$\angle DAE = \frac{\angle B - \angle C}{2}$

$= \frac{58° - 34°}{2} = \frac{24°}{2}$

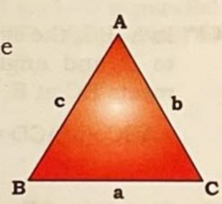
$= 12°$



☞ (i) Sum of any two sides of a triangle is always greater than the third side.

$a + b > c$, $b + c > a$

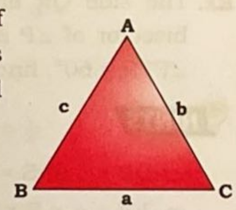
$c + a > b$



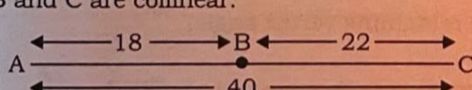
(ii) The difference of the length of any two sides of a triangle is always smaller than the third side.

$|a - b| < c$, $|b - c| < a$

$|c - a| < b$

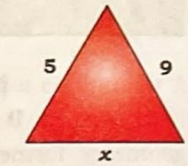


1. If one side of a triangle is greater than the sum of the other two sides then a triangle can't be formed.
2. When the length of one side is equal to sum of the length of other two sides then the points are collinear & triangle is not formed (just a straight line back and forth).i.e., if a + b = c then point A, B and C are collinear.



Ex. Three sides of a triangle are 5 cm, 9 cm, and x cm. The minimum integral value of x is.

HINTS



Value of x lies between, $4 < x < 14$

Thus, Minimum integral value of x is 5.

Ex. If the sides of a triangle are 7, 12 and x, and x is an integer, then find the number of possible values of x.

HINTS $12 + 7 > x > 12 - 7$

$\Rightarrow 19 > x > 5$

$x = (19 - 5) - 1 = 13$

Alternatively

No. of possible values of x = 2 × smallest side – 1

$= 2 \times 7 - 1 = 13$

"Number of Possible value of x = 2 × smallest side – 1"

Area of Triangle

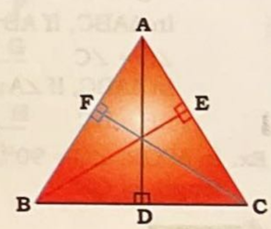
(A) Area of $\Delta ABC = \frac{1}{2} \times$ base × height

To calculate the area of ΔABC, we take any of the sides as base and the corresponding perpendicular from the vertex to the base as the height.

In ΔABC,
$AD \perp BC$, $BE \perp AC$ and
$CF \perp AB$.

$ar(\Delta ABC) = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times AC \times BE = \frac{1}{2} \times AB \times CF$



(i) If the height of the two triangles is equal, the ratio of their areas is proportional to the ratio of their bases.

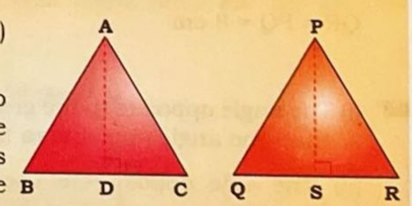
- If AD = PS, then

  Ar (ΔABC) : Ar (ΔPQR)
  = BC : QR

(ii) If the bases of the two triangles is equal, the ratio of their areas is proportional to the ratio of their height.

- If BC = QR, then

  Ar (ΔABC) : Ar (ΔPQR) = AD : PS



(B) Area of scalene triangle

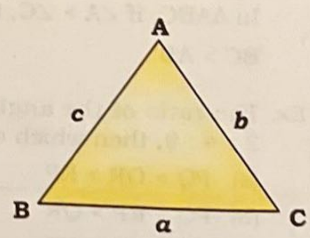
Area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$

(Heron's formula)

Where, Semi Perimeter (s)

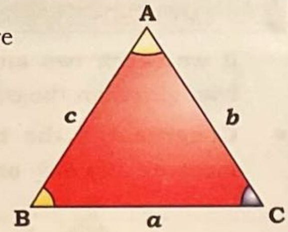
$= \frac{a+b+c}{2}$



(C) When two sides and the angle between these two sides are given

Area of $\Delta ABC = \frac{1}{2}$ ab sinC

$= \frac{1}{2}$bc sinA $= \frac{1}{2}$ac sinB



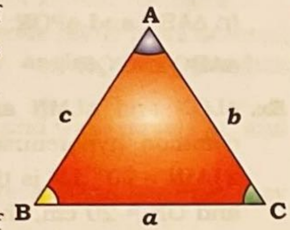
Sine & Cosine Rule

(a) Sine Rule

The ratio of side and sine of opposite angle of a triangle is equal to twice of circum radius.

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R;$

Where, R is the circumradius of triangle.



(b) Cosine Rule

If two sides and the angle between those sides are given, then we can find the opposite side by Cosine Rule.

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Ex. In ΔABC, $\angle B = 30°$ and $\angle C = 45°$. If BC = 50cm, then find the length of AB.

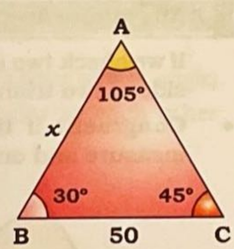
HINTS

$\Rightarrow \frac{50}{\sin 105°} = \frac{x}{\sin 45°}$

$\Rightarrow \frac{50}{\frac{(\sqrt{3}+1)}{2\sqrt{2}}} = \frac{x}{\frac{1}{\sqrt{2}}}$

$\left(\because \sin 105° = \frac{(\sqrt{3}+1)}{2\sqrt{2}}\right)$

$\Rightarrow x = \frac{100}{(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = 50(\sqrt{3}-1)$



Ex. In ΔABC, AB = 12cm and AC = 10cm, and $\angle BAC = 60°$. What is the length of side BC?

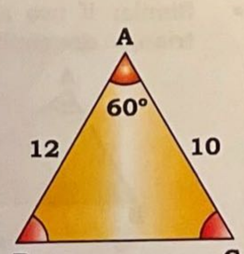
HINTS $\cos A = \frac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC}$

$\Rightarrow \cos 60° = \frac{(12)^2 + (10)^2 - BC^2}{2 \times 12 \times 10}$

$\Rightarrow \frac{1}{2} = \frac{144 + 100 - BC^2}{2 \times 120}$

$\Rightarrow 120 = 244 - BC^2$

$\Rightarrow BC^2 = 124 \Rightarrow BC = 2\sqrt{31}$ cm



Congruency & Similarity in Triangles

Congruency in Triangles

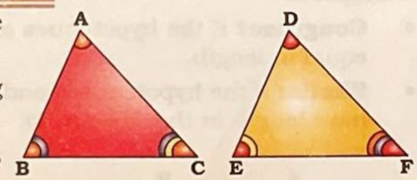
- Two triangles are Congruent, if

(i) Their corresponding angles are equal

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

(ii) Their corresponding sides are equal.

AB = DE, BC = EF, CA = FD



Similarity in Triangles

- Two triangles are similar, if

(i) Their corresponding angles are equal

$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

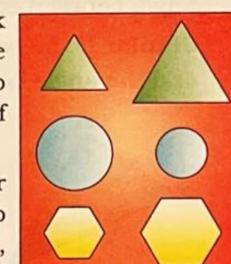
(ii) Their corresponding sides are in the same ratio (or proportion).

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

All congruent figures are similar, but not all similar figures are congruent.

Similarity in Regular Figures

Any two figures could be tested to check for similarity and congruence. In the case of regular figures, this is easiest – any two regular figures with the same number of sides will be similar to each other.



For example if we take two regular hexagons, or two circles, or two equilateral triangles, or two squares, or two regular pentagons, each pair of figures will be similar without any further checking required.

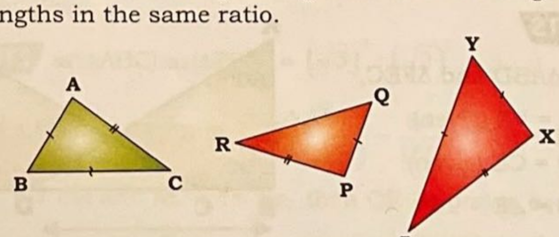
The figure may not always look similar-one should test to make sure.

Conditions for Congruency and Similarity of Triangles

SSS (Side-Side-Side) Test

If we check the three sides of two triangles, then the triangles are

- **Congruent** if three pairs of sides of the two triangles are equal in length.
- **Similar** if the corresponding sides of two triangles have lengths in the same ratio.



In ΔABC and ΔPQR: AB = PQ, BC = QR, AC = PR

$\therefore \Delta ABC \cong \Delta PQR$

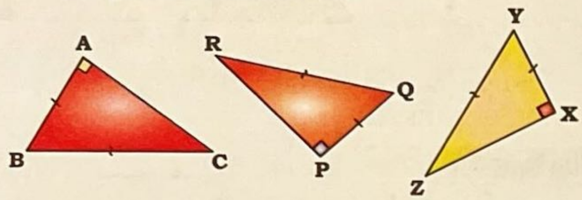
In ΔPQR and ΔXYZ: $\frac{PQ}{XY} = \frac{QR}{YZ} = \frac{PR}{XZ}$

$\therefore \Delta PQR \sim \Delta XYZ$

Hypotenuse Side Test

If we check the sides of two right-angled triangles, then the triangles are

- **Congruent** if the hypotenues and one shorter sides are equal in length.
- **Similar** if the hypotenuses and one pair of shorter sides have length in the same ratio.



In ΔABC and ΔPQR: $AB = PQ$, $BC = RQ$

$\therefore \Delta ABC \cong \Delta PQR$

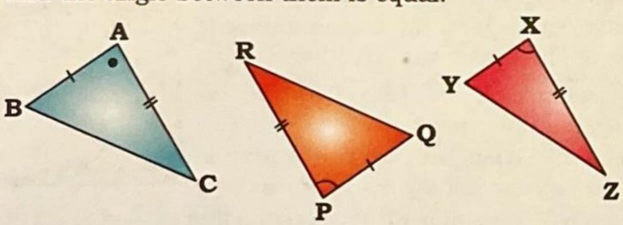
In ΔPQR and ΔXYZ: $\frac{PQ}{XY} = \frac{QR}{YZ}$

$\therefore \Delta PQR \sim \Delta XYZ$

SAS (Side-Angle-Side) Test

If we check two side and its included angle of two triangles then the triangle are

- **Congruent** if the two sides are equal in length and the angle between equal sides is also equal.
- **Similar** if the two sides have lengths in the same ratio and the angle between them is equal.



In ΔABC and ΔPQR: $AB = PQ$, $AC = PR$, $\angle A = \angle P$

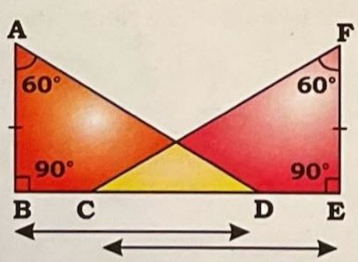
$\therefore \Delta ABC \cong \Delta PQR$

In ΔPQR and ΔXYZ: $\frac{PQ}{XY} = \frac{QR}{YZ}$, $\angle P = \angle X$

$\therefore \Delta PQR \sim \Delta XYZ$

Ex. In ΔABD and ΔFEC, $\angle BAD = 60°$, $l(BD) = l(EC)$, $\angle ABD = \angle FEC = 90°$, and $l(AB) = l(FE)$. Find the ratio of $\angle BAD$ and $\angle FCE$.

HINTS



In ΔABD and ΔFEC,

$AB = FE$ (given)

$BD = CE$ (given)

$\angle B = \angle E = 90°$

$\Delta ABD \cong \Delta FEC$ (from SAS)

$\Rightarrow \angle F = 60°$

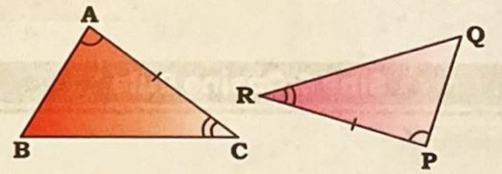
$\therefore \angle FCE = 180° - 90° - 60° = 30°$

$\therefore \frac{\angle BAD}{\angle FCE} = \frac{60°}{30°} = \frac{2}{1} = 2:1$

ASA (Angle-Side-Angle) Test

If we check two angles and the included side of two triangles then the triangle are

- **Congruent** if the two pairs of angle have the same measure and only one side are equal in length.

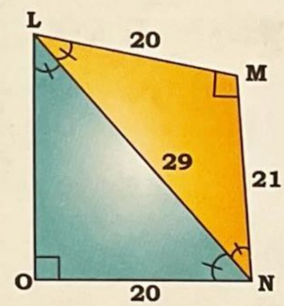


In ΔABC and ΔPQR: $BC = QR$, $\angle A = \angle P$, $\angle C = \angle R$

$\Delta ABC \cong \Delta PQR$

Ex. ΔLON and ΔLMN are two right-angled triangles with common hypotenuse LN such that $\angle LON = 90°$ and $\angle LMN = 90°$. LN is the bisector of $\angle OLM$. If LN = 29 cm and ON = 20 cm, then what is the perimeter (in cm) of ΔLMN?

HINTS



In ΔLON and ΔLMN,

$\angle LON = \angle LMN = 90°$

LN = LN (common side)

$\angle OLN = \angle MLN$ (angle bisector)

$\therefore \Delta LON \cong \Delta LMN$ (By ASA)

In ΔLMN,

$MN^2 = 29^2 - 20^2 \Rightarrow MN = 21$

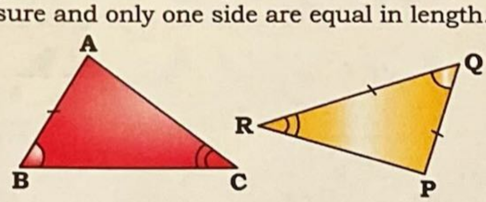
$\therefore$ Perimeter of $\Delta LMN = 29 + 20 + 21$

$= 70$ cm

AAS (Angle-Angle-Side) Test

If we check two angles and a corresponding non included side of two triangles then the triangle are

- **Congruent** if the two pairs of angles have the same measure and only one side are equal in length.



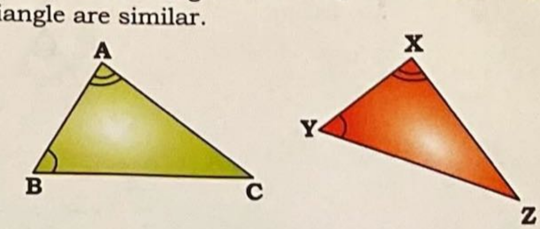
In ΔABC and ΔPQR: $AB = PQ$, $\angle B = \angle Q$, $\angle C = \angle R$

$\Delta ABC \cong \Delta PQR$

AA Test

If we check the angles of two triangles then the triangles are

- Similar if two angles of triangle are equal then both triangle are similar.

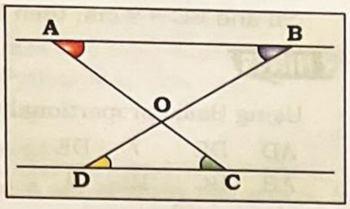


In ΔABC and ΔPQR: $\angle A = \angle X$, $\angle B = \angle Y$

$\Delta ABC \sim \Delta XYZ$

Spotting Similarity & Congruency

- Identifying similarity (and congruency) is crucial in geometry, especially for visualizing problems.
- A key to recognizing similarity is spotting equal angles.



Case 01: Parallel Lines

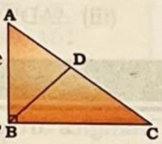
- When you see two parallel lines intersected by transversals, immediately look for similar triangles.
- This is because parallel lines create many pairs of equal angles (e.g., alternate interior angles, corresponding angles).

  Given parallel lines AB and CD, and transversals AC and BD intersecting at O:
- Triangles ΔAOB and ΔCOD are similar because they have two pairs of equal angles:
  - ∠OAB = ∠OCD (Alternate interior angles)
  - ∠OBA = ∠ODC (Alternate interior angles)
  - ∠AOB = ∠COD (Vertically opposite angles)

Case 02: Altitude to the Hypotenuse in a Right Triangle:

When you draw an altitude from the right angle vertex to the hypotenuse of a right-angled triangle, it creates three similar triangles:

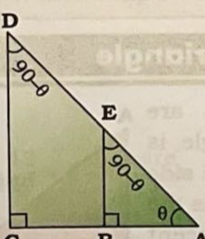
- The original right triangle.
- The two smaller triangles formed by the altitude.



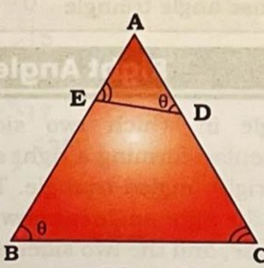
- Example (using ΔABC right-angled at B, with BD as altitude to AC):
- ΔABC ~ ΔADB (They share ∠A, and both have a right angle).
- ΔABC ~ ΔBDC (They share ∠C, and both have a right angle).
- Therefore, all three triangles are similar to each other: ΔABC ~ ΔADB ~ ΔBDC.

Some Similar Figure

-

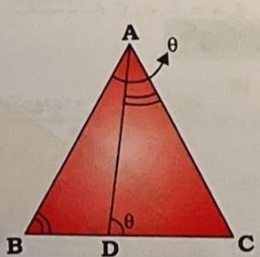


ΔABE ~ ΔACD

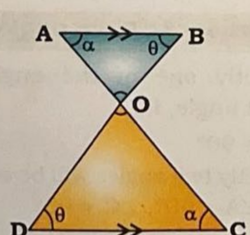


ΔADE ~ ΔABC

-



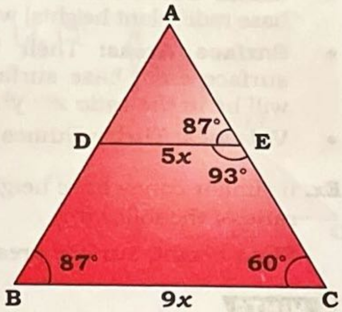
∠ADC = ∠BAC = θ
∠DAC = ∠ABC (Given)
ΔABC ~ ΔADC



ΔAOB ~ ΔCOD

Ex. In ΔABC, ∠B = 87° and ∠C = 60°. Points D and E are on the sides AB and AC, respectively, such that ∠DEC = 93° and DAE is

HINTS



∠DEA = ∠ABC

∠A = Common

⇒ ΔAED ~ ΔABC

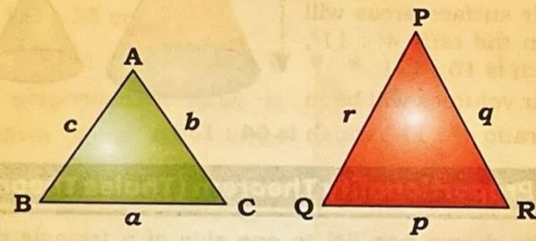
$$\Rightarrow \frac{AE}{AB} = \frac{ED}{BC} = \frac{AD}{AC}$$

$$\Rightarrow \frac{AE}{14.4} = \frac{5}{9}$$

⇒ AE = 8 cm

Properties in Similar triangles

If ΔABC and ΔPQR are similar, then



(i) $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$

(ii) Ratio of corresponding sides
= Ratio of perimeter
= Ratio of semi-perimeter(s)
= Ratio of corresponding medians
= Ratio of inradius
= Ratio of circumradius

(iii) Ratio of area
= Ratio of square of corresponding sides
= Ratio of square of perimeter
= Ratio of square of semi-perimeter
= Ratio of square of corresponding medians
= Ratio of square of inradius
= Ratio of square of circumradius

Ex. If the ratio of corresponding sides of two similar triangles is $\sqrt{3} : \sqrt{2}$, then what is the ratio of the area of the two triangles?

HINTS $ar(\Delta ABC):ar(\Delta PQR) = (\sqrt{3})^2 : (\sqrt{2})^2 = 3 : 2$

Ex. Let ΔABC ~ ΔQPR and $\frac{ar(\Delta ABC)}{ar(\Delta QPR)} = \frac{64}{169}$. If AB = 10 cm, BC = 7 cm and AC = 16 cm, then QR (in cm) is equal to:

HINTS Here, ΔABC ~ ΔQPR & $\frac{ar(\Delta ABC)}{ar(\Delta QPR)} = \frac{64}{169}$

$$\Rightarrow \frac{AC}{QR} = \sqrt{\frac{64}{169}} \Rightarrow \frac{16}{QR} = \frac{8}{13}$$

$$\Rightarrow QR = \frac{13 \times 16}{8} = 26 \text{ cm}$$

Similarity of Cones

If two cones are similar and their heights (a linear dimension) are in the ratio $x : y$.

- **Linear Dimensions:** Their other linear dimensions (like base radii, slant heights) will also be in the same ratio $x : y$.
- **Surface Areas:** Their various surface areas (total surface area, base surface area, curved surface area) will be in the ratio $x^2 : y^2$.
- **Volumes:** Their volumes will be in the ratio $x^3 : y^3$.

Ex. If similar cones have heights in the ratio 4 : 11. Find the ratio of the following:

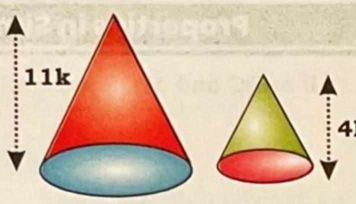
Slant height, surface area and volume

HINTS

Their base radii and slant heights will also be in the ratio 4 : 11.

Their surface areas will be in the ratio $4^2 : 11^2$, which is 16 : 121.

Their volumes will be in the ratio $4^3 : 11^3$, which is 64 : 1331.



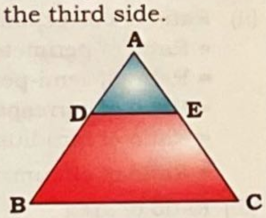
Basic Proportionality Theorem (Thales Theorem)

- A line drawn parallel to one side of a triangle divides other two sides in the same ratio.
- If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

In ΔABC, If $DE \parallel BC$, then

$$\frac{AD}{DB} = \frac{AE}{EC}$$

'OR' If $\frac{AD}{DB} = \frac{AE}{EC}$, then $DE \parallel BC$



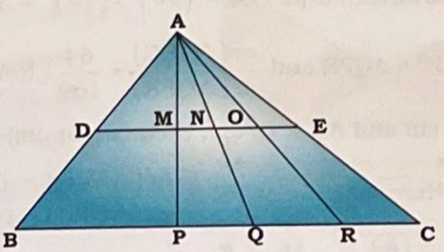
- Some of the results derived from this theorem, we will use, are as follows:

(i) $\frac{AD}{BD} = \frac{AE}{EC}$ (ii) $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

(iii) $\Delta ADE \sim \Delta ABC$

(iv) $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{AE}{AC}\right)^2 = \left(\frac{DE}{BC}\right)^2$

- A line drawn parallel to one side of a triangle divides the median, the angle bisector and the altitude of triangle in the sain ratio it divides the other two sides of the triangle.



In ΔABC, AP, AQ and AR are the median, the angle bi-sector and altitude respectively and $DE \parallel BC$, then

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{AM}{MP} = \frac{AN}{NQ} = \frac{AO}{OR}$$

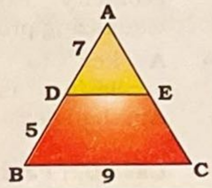
Ex. In ΔABC, D and E are the points on sides AB and AC, respectively such that $\angle ADE = \angle B$. If AD = 7 cm BD = 5 cm and BC = 9 cm, then DE (in cm) is equal to____

HINTS

Using Basic proportionality theorem,

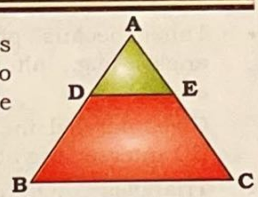
$$\frac{AD}{AB} = \frac{DE}{BC} \Rightarrow \frac{7}{12} = \frac{DE}{9}$$

$$\Rightarrow DE = \frac{63}{12} = 5.25 \text{ cm}$$



Mid-Point Theorem

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of the magnitude of third side and vice-versa.



- If D and E are mid-points of AB and AC, respectively, then $DE \parallel BC$ and $DE = \frac{BC}{2}$
- $DE \parallel BC$ and $DE = \frac{BC}{2}$, then D and E are the mid-points of AB and AC respectively. In this case

(i) $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} = \frac{1}{2}$ (ii) $\frac{AD}{DB} = \frac{AE}{EC} = 1$

(iii) $\Delta ADE \sim \Delta ABC$ (iv) $\frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{1}{4}$

Types of Triangle

Triangles are classified on the basis of angles and sides–

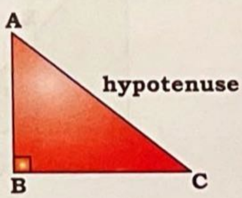
Types of Triangle

On the basis of Angle
- Right angle triangle
- Acute angle triangle
- Obtuse angle triangle

On the basis of Sides
- Scalene triangle
- Isosceles triangle
- Equilateral triangle

Right Angle Triangle

A triangle in which two sides are perpendicular, forming a right angle, is called a right-angled triangle. The side opposite the right angle is known as the hypotenuse, and the two sides adjacent to the right angle are called legs.



Properties of a Right angle Triangle

(i) Exactly one of the angle is right angle, i.e.

$\angle B = 90°$

- Exactly two angles will be acute. i.e. $\angle A < 90°$, $\angle C < 90°$
- One angle is equal to the sum of other two angle, i.e.

$\angle B = \angle A + \angle C = 90°$