CHAMPION PUBLICATION

English Medium

4th Edition

# MATHS

# Concept King

## Formula Book

- All formulae and theorems with examples
- Smart tricks with exam hall approach

Arithmetic & Advance Maths

*Useful For*

SSC-CGL (Tier-I & II), CHSL(Tier-I & II), CPO, MTS, CDS, GD, Selection Post, Railway & other Competitive Exams

*Author*

**Gagan Pratap Sir**

*Co-Author & Editor*

**Manvendra Singh**

# INDEX

# INDEX

# Geometry (Line & Angle)

## Line and Angle

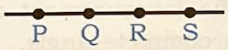
◈ **Point (•):→** It is a zero dimensional figure or a circle with zero radius.

### Types of point

**(i) Collinear point:→**

If 3 or more than 3 points lie on a line close to or far from each other, then they are said to be collinear.

**Ex.** Points P, Q, R, S are collinear




**(ii) Non-collinear point:→**

If 3 or more points are not situated on a straight line, these all points are called non-collinear points.

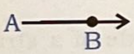
**Ex.**

## Types of lines

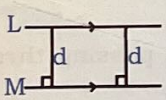
◈ **Line (One dimensional figure):→** It is a set of points having only length with no ends.

◈ **Line segment:→** A line with a fixed length is called a line segment.


◈ **Ray:→** A line with unidirectional length is called a ray. It is a line with single endpoint that extends infinitely in one direction.

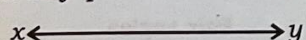



◈ **Parallel lines:→** Two or more lines that never intersect each other are called parallel lines.

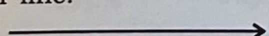



In this figure L is parallel to M and is denoted by L ∥ M.

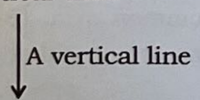
◈ **Straight line:→** A line that does not change its direction at any point is called a straight line.



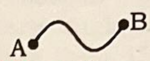

◈ **Horizontal lines:→** When a line moves from left to right or right to left in a straight direction, it is a horizontal line.



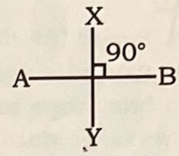
◈ **Vertical lines:→** When a line runs from top to bottom or bottom to top in a straight direction, it is called a vertical line.




◈ **Curved line:→** A line which is not a straight line or a line which changes its directions is called a curved line.



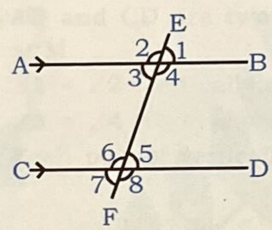

◈ **Perpendicular line:→** If two lines intersect at 90°, then these lines are called perpendicular lines.



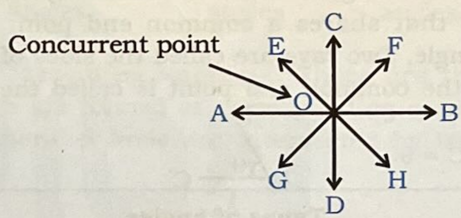

In this figure AB and XY are perpendicular lines and is denoted by AB ⊥ XY or XY ⊥ AB.

◈ **Transversal Line:→** A line which intersects (touches) two or more lines at distinct point is called transversal lines of the given lines.

AB ∥ CD and EF is transversal line.

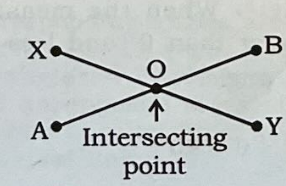



◈ **Concurrent line:→** Three or more than three lines which pass from a single point are called concurrent lines.




AB, CD, GF, EH are concurrent lines and O is the concurrent point.

◈ **Intersecting line:→** If two or more lines intersect each other, then they are called intersecting lines.




**Extra facts about lines:→**

◈ The intersection of two planes is a line.

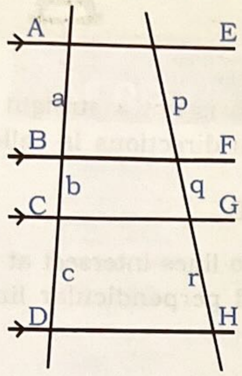
◈ It has no ends in both directions.

◈ It has no thickness.

◈ It is one-dimensional figure.

## Some important results

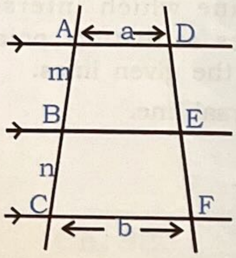
- When AE || BF || CG || DH and these lines are cut by two transversal lines.




then a : b : c = p : q : r

$$\frac{a}{a+b+c} = \frac{p}{p+q+r}$$

- When AD || BE || CF and these lines are cut by two transversal lines.



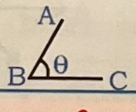

then $\frac{AB}{BC} = \frac{DE}{EF} = \frac{m}{n}$

$$BE = \frac{an+bm}{m+n}$$

## Types of angles

- **Angle:→** A figure which is formed by two rays or lines that shares a common end point is called an angle. Two rays are called the sides of an angle and the common end point is called the vertex.

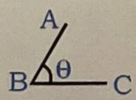
$\angle ABC = \theta$




**1.** **Types of angles**

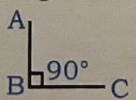
- Acute angle
- Right angle
- Obtuse angle
- Straight angle
- Reflex angle
- Complete angles

**(i) Acute angle:→** When the measurement of an angle is greater than 0° and less than 90°, it is called acute angle.



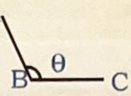

$0° < \theta < 90°$

**(ii) Right angle:→** When the measurement of an angle is exactly 90°, it is called a right angle.




$AB \perp BC$ and $\angle ABC = 90°$

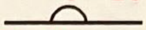
**(iii) Obtuse angle:→** When the measurement of an angle is greater than 90° but less than 180°, it is called an obtuse angle.



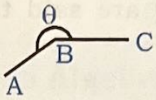

$90° < \theta < 180°$

**(iv) Straight angle:→** When the measurement of an angle is 180°, it is called a straight angle or line angle.

$\theta = 180°$

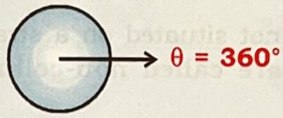


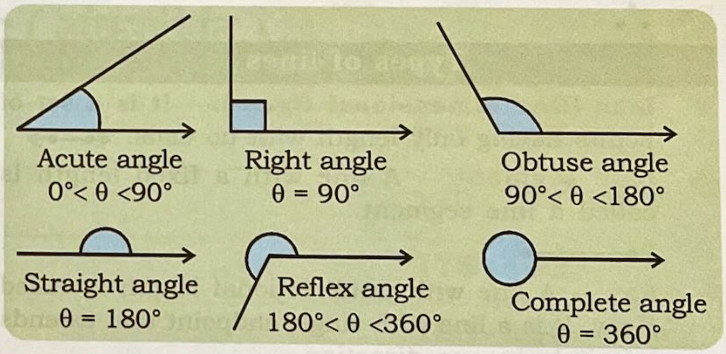
**(v) Reflex angle:→** When the measurement of an angle is greater than 180° but less than 360°, it is called a reflex angle.




$180° < \theta < 360°$

**(vi) Complete angle:→** When the measurement of an angle is 360°, it is called a complete angle.




**Extra facts about angle:→**

**(i)** The sum of all the angles on one side of a straight line is always equal to 180°.

**(ii)** The sum of all the angles around a point is always equal to 360°.

**(iii)** There is one and only one line passing through two distinct point.

Two or more line are said to be co-planar if they lie in the same plane, otherwise they are said to be non-coplanar.

**2.** **Pair angles**

- Adjacent angles
- Linear pair angles
- Complementary angles
- Supplementary angles

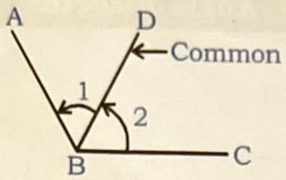
**(i) Adjacent angles:→**

Two angles are said to be adjacent if

(a) They have a common vertex (vertex B)

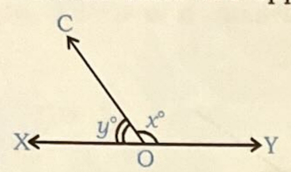
(b) They have a common arm. (BD is common)

(c) Uncommon arms are on either side of the common arm.




In the above figure ∠1 and ∠2 are adjacent angles. They have common vertex B and common arm BD and two uncommon arms BA and BC are on the opposite side of the line BD.

**(ii) Linear pair angle:→** It is a pair of adjacent angle whose non-common sides are opposite rays.




$\angle x + \angle y = 180°$

⇒ Linear pair angles are supplementary.

◈ In the above figure OX and OY are opposite rays. Therefore, ∠XOC and ∠YOC are adjacent angles which forms a linear pair.

◈ If two opposite lines meet, the sum of adjacent angles so formed is 180°.

**Ex.** Determine the other angle of a linear pair if:

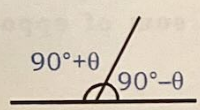
(a) one of its angles is acute?

(b) one of its angles is obtuse?

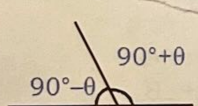
(c) one of its angles is right?

**Sol.** A linear pair is a pair of adjacent angles that are supplementary, meaning their sum is 180°.

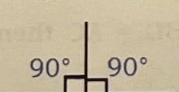
(a) If one angle is acute (less than 90°), the other angle of a linear pair is obtuse (more than 90°).




(b) If one angle is obtuse, the other angle of a linear pair is acute.

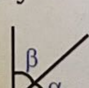



(c) If one angle is a right angle (90°), the other angle of a linear pair is also a right angle.




**(iii) Complementary Angle:→** If sum of two angles is 90° then they are complementary to each other.

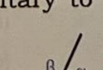
Complementary Angle




$\alpha+\beta = 90°$

**(iv) Supplementary Angle:→** If sum of two angles is 180° then they are supplementary to each other.

Supplementary Angles




$\alpha+\beta = 180°$

| Angle | Complementary | | Supplementary |
|---|---|---|---|
| 43° | 47° | —+90→ | 137° |
| 12° | 78° | —+90→ | 168° |
| θ | 90° − θ | —+90→ | 180° − θ |

**Note:-** Supplementary angle of an angle is 90° more than its complementary angle.

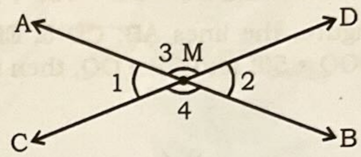
**Ex.** An angle is more than 45°. What can you say about its complementary angle? Is it more than 45°, equal to 45° or less than 45°?

**Sol.** Let $\theta_1$ and $\theta_2$ two complementary angles

$\therefore\ \theta_1 + \theta_2 = 90°$

If $\theta_1$ is more than 45° then $\theta_2$ must be less than 45°.

**3. Vertically opposite angles:→** These are the angles which are equal and opposite to each other at a vertex which is created by two straight intersecting lines.




AB and CD are two intersecting lines, intersect at M.

∠1 = ∠2 (vertically opposite angles)

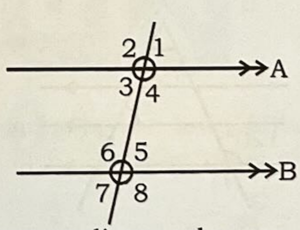
∠3 = ∠4 (vertically opposite angles)

◈ Each pair of vertically opposite angles are equal.

**4.** Angles made by a transversal line

- Corresponding angles
- Alternate angles
- Interior angles

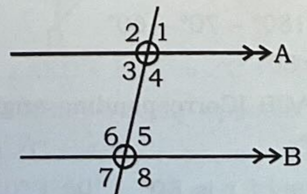
**(i) Corresponding angles:→** These are the angles which are formed at corresponding corners when two parallel lines are intersected by transversal line.




Pair of corresponding angles are

∠1 = ∠5, ∠4 = ∠8, ∠2 = ∠6, ∠3 = ∠7

**(ii) Alternate angles:→** These are the angles which are formed on opposite sides of the transversal line when two parallel lines are intersected by that transversal line.

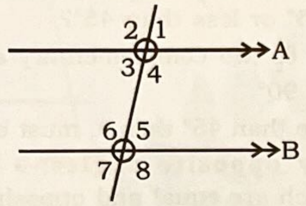


**Pair of alternate angles:**

Here, interior alternate angles are $\angle 3 = \angle 5$, $\angle 4 = \angle 6$

exterior alternate angles are $\angle 1 = \angle 7$, $\angle 2 = \angle 8$

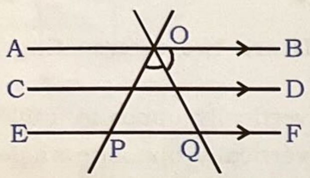
◈ **Interior angles:→** Two pairs of interior angles are formed on either side of transversal when two parallel lines are cut by a transversal line. The pairs of interior angles so formed are supplementary i.e., their sum is 180°.



Pair of interior angles:

$\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$.

**Ex.1** In the figure, the lines AB, CD & EF are parallel lines. $\angle BOQ = 50°$ and OP = OQ, then find the value of $\angle POQ$.



(a) 50° (b) 60°

(c) 100° (d) 80°

**Sol. (d)** $\angle BOQ = 50°$

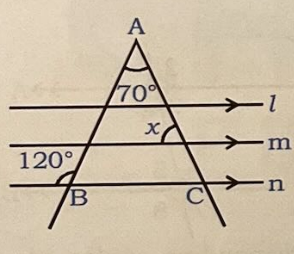
$\therefore \angle OQP = \angle BOQ = 50°$ (Alternate angles)

Since OP = OQ

$\therefore \angle OPQ = \angle OQP = 50°$

$\therefore \angle POQ = 180° - 50° - 50° = $ **80°**

**Ex.2** In the given figure $l \parallel m \parallel n$, then value of $x$ is:



(a) 80° (b) 70°

(c) 60° (d) 50°

**Sol. (d)** $\angle ABC = 180° - 120° = 60°$

$\Rightarrow \angle ACB = 180° - \angle A - \angle ABC$

$\Rightarrow \angle ACB = 180° - 70° - 60°$

$\Rightarrow \angle ACB = 50°$

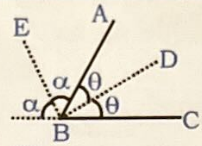
$\Rightarrow \angle x = \angle ACB$ [Corresponding angles]

$\Rightarrow \angle x = 50°$

$\therefore$ The value of $x$ is **50°**.

## Angle Bisector

◈



BE → exterior angle bisector of $\angle ABC$

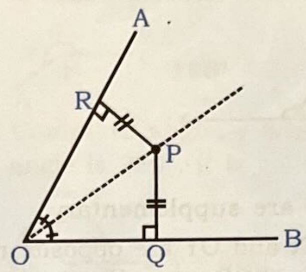
$2\alpha + 2\theta = 180°$

$\alpha + \theta = 90° = \angle EBD$

$\therefore$ Angle between internal angle bisector and external angle bisector of an angle is 90°.

BD is interior angle bisector of $\angle ABC$.

◈

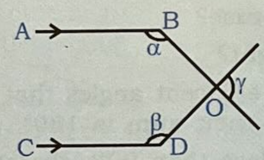


P is any point on angle bisector of $\angle AOB$.

**PR = PQ** (Result)

## Some Useful Results

◈ **If AB ∥ CD then find the value of $\alpha+\beta+\gamma$?**

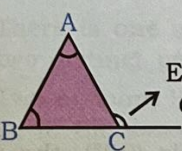


$\alpha + \beta + \gamma = 360°$

◈ **Exterior angle is equal to sum of opposite interior angles.**

$\angle A + \angle B + \angle C = 180°$

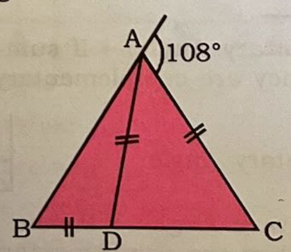
$\angle A + \angle B = 180° - \angle C$



Exterior angle at vertex C = $180° - \angle C = \angle A + \angle B$

sum of all exterior angles = 360°

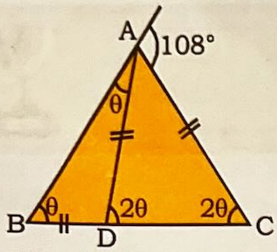
**Ex.** In the given triangle, if AD = BD = AC then the value of angle C will be?



(a) 72° (b) 90°

(c) 54° (d) 64°

**Sol.(a)**



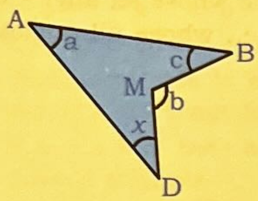

$\angle ADC = \theta + \theta = 2\theta$ (exterior angle)

$AD = AC \therefore \angle ADC = \angle ACD = 2\theta$

$\therefore \theta + 2\theta = 108°$ (exterior angle)

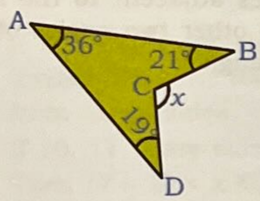
$\theta = 36° \therefore \angle ACB = 2\theta =$ **72°**

◈ **In the figure, ABMD is a quadrilateral.**



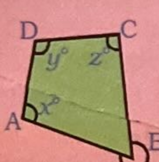

**b = a + c + x**

**Ex.** In the given figure, find the value of $x$.




**$x° = 36° + 21° + 19° = 76°$**

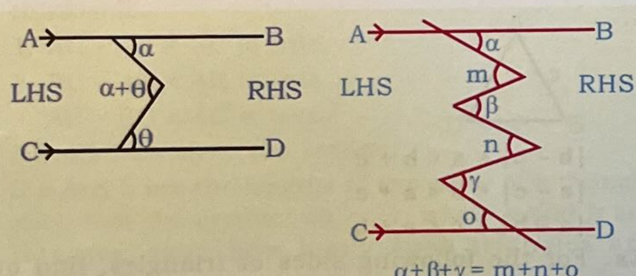
◈ **In the given figure, ABCD is a quadrilateral.**




**∠B (internal) = 360° – (x°+y°+z°)**

**∠B (external) = x°+y°+z°**

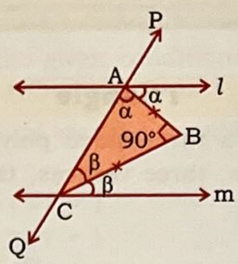
◈




**Sum of angle on RHS = Sum of angle on LHS**

$\alpha+\theta = \alpha+\theta$

**Ex.** In the given figure line $l$ and m are parallel to each other and AB and CB are the angle bisector of ∠A and ∠C respectively then ∠ABC = 90°.



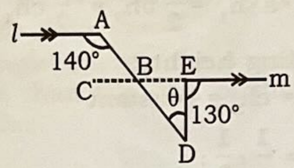

**Sol.** $2\alpha + 2\beta = 180°$ {interior angle of same side}

$\alpha + \beta = 90°$

$\alpha + \beta + \angle ABC = 180°$

$\angle ABC =$ **90°**

**Ex.** In the given figure line $l$ and m are parallel to each other then find the value of θ?




**Sol.** $\angle A + \angle ABC = 180°$ {Interior angle of same side}

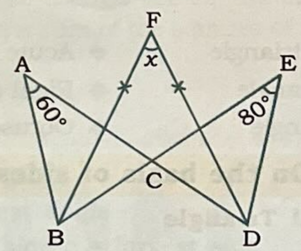
$\angle ABC = 40° = \angle EBD$ (Vertically opposite angle)

$\Rightarrow \angle EBD + \theta = 130° \Rightarrow \theta = 130° - 40° =$ **90°**

**Trick:→** $\theta = (\angle A + \angle E) - 180° = (140° + 130°) - 180°$

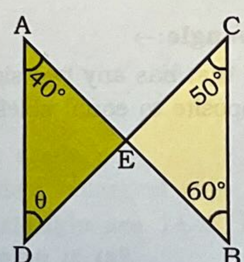
$\theta = 270° - 180° =$ **90°**

**Ex.** In the given figure BF and DF are the angle bisector of ∠ABC and ∠EDC respectively then find the value of $x$?




**Sol.** $x = \dfrac{60° + 80°}{2} =$ **70° {Concept}**

**Ex.** In the given figure two lines AB and CD itersect each other at E. Find the value of θ?




**Sol.** $\angle A + \angle D + \angle AED = 180° \Rightarrow \angle C + \angle B + \angle CEB = 180°$

$\angle AED = \angle CEB$ {Vertically opposite angle}

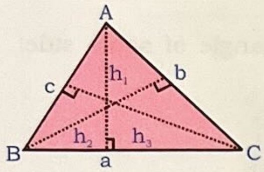
$\angle A + \angle D = \angle C + \angle B$ {Concept}

$40° + \theta = 50° + 60° \Rightarrow \theta =$ **70°**

# Types of Triangles

## Triangle

◈ A triangle is a three-sided polygon that consists of three edges, three vertices, three altitudes and three angles.




$\angle A + \angle B + \angle C = 180°$

Area $\Rightarrow \frac{1}{2} \times a \times h_1 = \frac{1}{2} bh_2 = \frac{1}{2} ch_3 = \frac{1}{2} \times$ Base $\times$ Corresponding height.

$\Rightarrow ah_1 = bh_2 = ch_3 =$ constant

$h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$

**OR**

$a : b : c = \frac{1}{h_1} : \frac{1}{h_2} : \frac{1}{h_3}$

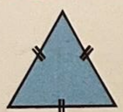
**Types of triangles**

On the basis of sides
- Equilateral triangle
- Isosceles triangle
- Scalene triangle

On the basis of angles
- Acute angle triangle
- Right angle triangle
- Obtuse angle triangle

## On the basis of sides

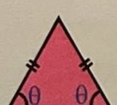
◈ **Equilateral Triangle**



It has three equal sides and each angle is equal to 60°.

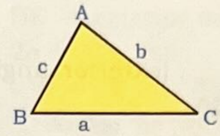
◈ **Isosceles Triangle:→**

It is a triangle that has any two sides equal in length and angles opposite to equal sides are also equal.




◈ **Scalene Triangle:→**

It can be defined as a triangle whose all three sides have different lengths and all the three angles are different.

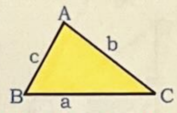



$\angle A \neq \angle B \neq \angle C$ & $a \neq b \neq c$

## On the basis of Angles

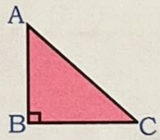
◈ **Acute Angle Triangle:→**

It is defined as the triangle whose all angles must be between 0° and 90° i.e., whose all angles are acute angles.



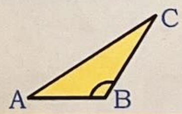

◈ **Right Angle Triangle:→**

It is a triangle in which two sides are perpendicular, forming a right angle (90°) between them. The side opposite to the right angle is called the hypotenuse. The sides adjacent to the right angle are called legs. The other two angles of the right triangle are acute angles.




◈ **Obtuse Angle Triangle:→**

It is defined as the triangle which have one obtuse angle (greater than 90°) and two acute angles. A triangle can not have more than one obtuse angle.

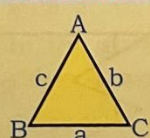



## Inequality of triangle

◈ The triangle inequality states that for any triangle the sum of the length of any two sides must be greater than the length of the third side.

◈ **Conditions for formation of triangle:→**

**1.**




$|b - c| < a < b + c$

$|a - c| < b < a + c$

$|a - b| < c < a + b$

**Ex.** For the following sides of triangles, find out if triangle is possible or not?

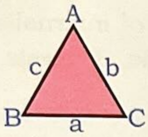
(i) 4, 9, 15

(ii) 5, 10, 15

(iii) 7, 12, 15

**Sol.** 4, 9, 15 Δ not possible $\because 4 + 9 < 15$
5, 10, 15 Δ not possible $\because 5 + 10 = 15$
7, 12, 15 Δ is possible $\because 7 + 12 > 15$
OR 7+15 > 12 OR 12 + 15 > 7 OR 15 – 12 < 7

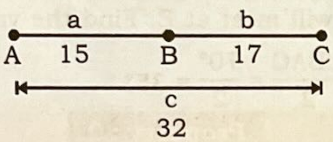
**2.** Sum of any two sides is always greater than 3rd side.




$a+b > c$, $b+c > a$, $c+a > b$

**3.** Difference of any two sides is always less than third side.
$\Rightarrow b > c - a$ $\Rightarrow b > a - c$
$\Rightarrow \therefore |c - a| < b < c + a$

**4.** When one side is equal to the sum of other two sides, it does not form a triangle.
If a + b = c, then a, b, c are collinear and do not form a triangle.




**Ex.** If 10, 17, $x$ are sides of a triangle and $x$ is an integer. Find the total number of triangles possible from these sides.

**Sol.** If 10, 17, $x$ are sides of a Δ, $x \rightarrow$ integer
Then $17 - 10 < x < 10 + 17 \Rightarrow 7 < x < 27$
$\therefore x \rightarrow \{8, 9, 10, .... 26\} \Rightarrow x_{min} = 8, x_{max} = 26$
$x_{total}$ = 19 values possible
∴ **19** Δ's possible

**Note:- Possible values of $x$ = 2×small side –1**
$\Rightarrow 2\times10-1 = 19$

**Ex.** Consider the following inequalities in respect of any triangle ABC:
1. AC – AB < BC
2. BC – AC < AB
3. AB – BC < AC

Which of the above are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

**Sol.** Difference of two sides of any triangle is less than third side.
1. AC – AB < BC is true.
2. BC – AC < AB is true.
3. AB – BC < AC is true.

So, all statement are correct.

**Ex.** If a and b are the lengths of two sides of a triangle such that the product ab = 24, where a and b are integers, then how many such triangles are possible?
(a) 16 (b) 15
(c) 18 (d) 12

**Sol. (b)** It is given that ab = 24, we can have different possible values of a and b such that:
$\Rightarrow 1 \times 24 : (24 - 1) < c < (24 + 1), c \Rightarrow 24$
$\Rightarrow 2 \times 12 : (12 - 2) < c < (12 + 2), c \Rightarrow 11, 12, 13$
$\Rightarrow 3 \times 8 : (8 - 3) < c < (8 + 3), c \Rightarrow 6, 7, 8, 9, 10$
$\Rightarrow 4 \times 6 : (6 - 4) < c < (6 + 4), c \Rightarrow 3, 4, 5, 6, 7, 8, 9$

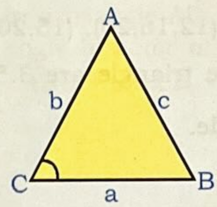
By studying the given conditions, the third side, c can take 16 different values.
∴ The number of triangles possible for ab = 24 is **16**.

**Alternatively:-**
$1 \times 24 \Rightarrow 2 \times 1 - 1 = 1$
$2 \times 12 \Rightarrow 2 \times 2 - 1 = 3$
$3 \times 8 \Rightarrow 2 \times 3 - 1 = 5$
$4 \times 6 \Rightarrow 2 \times 4 - 1 = 7$
∴ 1 + 3 + 5 + 7 = **16** triangles possible

## Relation between 3 sides of Triangle

◈ In any triangle the side opposite to the largest angle will be largest and the side opposite to the smallest angle is smallest.

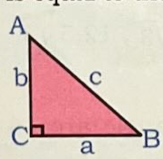
**I. Acute Angle Triangle:→**
In acute angle triangle the square of the longest side is less than the sum of the squares of two smaller sides.




$c^2 < a^2 + b^2$

∠C = largest angle
side AB = side c = largest side

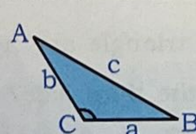
**II. Right Angle Triangle:→**
In right angle triangle the square of the longest side is equal to the sum of the squares of two smaller sides.




$c^2 = a^2 + b^2$

∠C = largest angle
side AB = side c = largest side

**III. Obtuse Angle Triangle:→**
In Obtuse angle triangle the square of the longest side is greater than the sum of the squares of two smaller sides.




$c^2 > a^2 + b^2$

∠C = largest angle
side AB = side c = largest side

**Ex. If sides of triangle are 11.7, 16.9, 23.4. Which type of triangle it is?**

**Sol.** Take ratio of sides 11.7 : 16.9 : 23.4
9 : 13 : 18
$18^2 > 9^2 + 13^2$
∴ Triangle is scalene and obtuse angle triangle.