CHAMPION
PUBLICATION

5th
Edition
ENGLISH
Medium

MATHS

Concept King

100 % Error Free

With VIDEO Solution

Formula Book

ORIGINAL PRODUCT
BEST SELLER
BEST QUALITY

20 Lakh+Students First Choice

- **Updated Eduquity Pattern**
- **All formulae and theorems with examples**
- **Smart tricks with exam hall approach**

Arithmetic & Advance Maths

Useful For

SSC-CGL (Tier-I & II), CHSL(Tier-I & II), CPO, MTS, CDS, GD, Selection Post, Railway & other Competitive Exams

Gagan Pratap Sir

Index

Index

01 Geometry (Line & Angle)

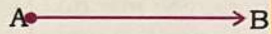
(Point) (•)	(Collinear points) P Q R S	(Non-collinear points)	(Intersecting Point)
It is a zero dimensional figure or a circle with zero radius.	If 3 or more than 3 points lie on a line close to or far from each other, then they are said to be collinear.	If 3 or more points are not situated on a straight line, these all points are called non-collinear points.	The common point where two or more lines, curves, or geometric figures intersect or meet.

Types of Lines

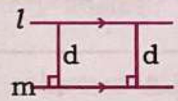
◈ **Line:→** It is a set of points having only length with no ends.

◈ **Line segment:→** A line with a fixed length is called a line segment.

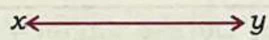
◈ **Ray:→** A line with unidirectional length is called a ray. It is a line with single endpoint that extends infinitely in one direction.



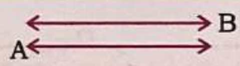
◈ **Parallel Lines:→** Two or more lines that never intersect each other are called parallel lines. In this figure l is parallel to m and is denoted by $l \parallel m$.



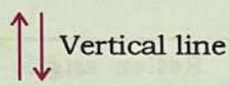
◈ **Straight Line:→** A line that does not change its direction at any point is called a straight line.



◈ **Horizontal Lines:→** When a line moves from left to right or right to left in a straight direction, it is a horizontal line.

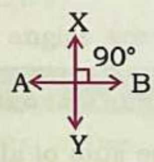


◈ **Vertical Lines:→** When a line runs from top to bottom or bottom to top in a straight direction, it is called a vertical line.

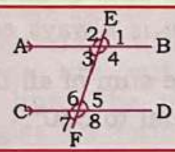


◈ **Curved Line:→** A line which is not a straight line or a line which changes its directions is called a curved line.

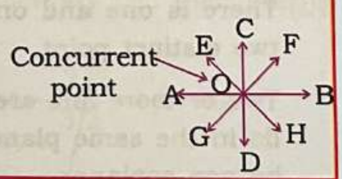
◈ **Perpendicular Line:→** If two lines intersect at 90°, then these lines are called perpendicular lines. In this figure AB and XY are perpendicular lines and is denoted by $AB \perp XY$ or $XY \perp AB$.



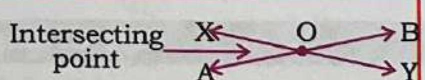
◈ **Transversal Line:→** A line which intersects (touches) two or more lines at distinct point is called transversal lines of the given lines. $AB \parallel CD$ and EF is transversal line.



◈ **Concurrent Line:→** Three or more than three lines which pass from a single point are called concurrent lines. AB, CD, GF, EH are concurrent lines and O is the concurrent point.



◈ **Intersecting Line:→** If two or more lines intersect each other, then they are called intersecting lines.



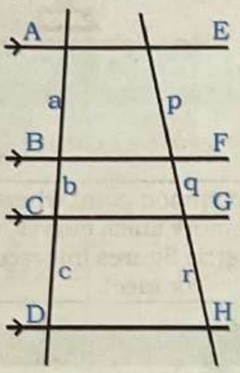
Extra facts about lines:→

◈ The intersection of two planes is a line.

◈ It has no ends in both directions. It is one-dimensional figure.

Some important results

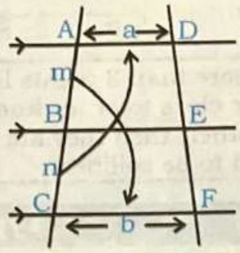
- When AE ∥ BF ∥ CG ∥ DH and these lines are cut by two transversal lines.



then a : b : c = p : q : r

$$\frac{a}{a+b+c} = \frac{p}{p+q+r}$$

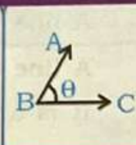
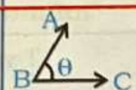
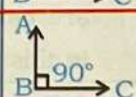
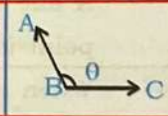
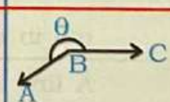
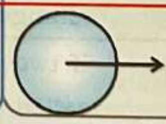
- When AD ∥ BE ∥ CF and these lines are cut by two transversal lines.



then $\frac{AB}{BC} = \frac{DE}{EF} = \frac{m}{n}$

$$BE = \frac{an + bm}{m + n}$$

Types of angles

Angle:→	A figure which is formed by two rays or lines that shares a common end point is called an angle. Two rays are called the sides of an angle and the common end point is called the vertex.		$\angle ABC = \theta$
(i) Acute angle:→	When the measurement of an angle is greater than 0° and less than 90°, it is called acute angle.		$0° < \theta < 90°$
(ii) Right angle:→	When the measurement of an angle is exactly 90°, it is called a right angle.		$AB \perp BC$, $\angle ABC = 90°$
(iii) Obtuse angle:→	When the measurement of an angle is greater than 90° but less than 180°, it is called an obtuse angle.		$90° < \theta < 180°$
(iv) Straight angle:→	When the measurement of an angle is 180°, it is called a straight angle or line angle.	$\theta = 180°$ 	
(v) Reflex angle:→	When the measurement of an angle is greater than 180° but less than 360°, it is called a reflex angle.		$180° < \theta < 360°$
(vi) Complete angle:→	When the measurement of an angle is 360°, it is called a complete angle.		$\theta = 360°$

Extra facts about angle:→

(i) The sum of all the angles on one side of a straight line is always equal to 180°.

(ii) The sum of all the angles around a point is always equal to 360°.

(iii) There is one and only one line passing through two distinct point.

Two or more line are said to be co-planar if they lie in the same plane, otherwise they are said to be non-coplanar.

Pair of angles

- Adjacent angles
- Linear pair angles
- Complementary angles
- Supplementary angles

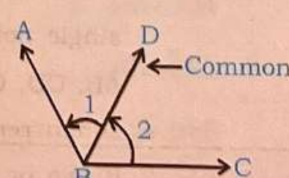
(i) Adjacent angles:→

Two angles are said to be adjacent if

(a) They have a common vertex (vertex B)

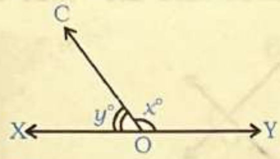
(b) They have a common arm. (BD is common)

(c) Uncommon arms are on either side of the common arm.



In the above figure ∠1 and ∠2 are adjacent angles. They have common vertex B and common arm BD and two uncommon arms BA and BC are on the opposite side of the line BD.

(ii) Linear pair angle:→ It is a pair of adjacent angle whose non-common sides are opposite rays.



$\angle x + \angle y = 180°$

⇒ Linear pair angles are supplementary.

- In the above figure OX and OY are opposite rays. Therefore, ∠XOC and ∠YOC are adjacent angles which forms a linear pair.
- If two opposite lines meet, the sum of adjacent angles so formed is 180°.

Exp. Determine the other angle of a linear pair if:

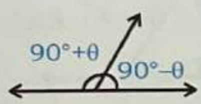
(a) one of its angles is acute?

(b) one of its angles is obtuse?

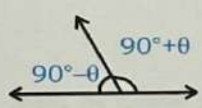
(c) one of its angles is right angle?

Idea A linear pair is a pair of adjacent angles that are supplementary, meaning their sum is 180°.

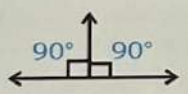
(a) If one angle is acute (less than 90°), the other angle of a linear pair is obtuse (more than 90°).



(b) If one angle is obtuse, the other angle of a linear pair is acute.

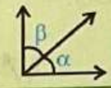


(c) If one angle is a right angle (90°), the other angle of a linear pair is also a right angle.



(iii) Complementary Angle:→ If sum of two angles is 90° then they are complementary to each other.

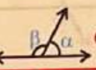
Complementary Angle



α+β = 90°

(iv) Supplementary Angle:→ If sum of two angles is 180° then they are supplementary to each other.

Supplementary Angles



α+β = 180°

Angle	Complementary	Supplementary
43°	47° $\xrightarrow{+90}$	137°
12°	78° $\xrightarrow{+90}$	168°
θ	90° − θ $\xrightarrow{+90}$	180° − θ

Note Supplementary angle of an angle is 90° more than its complementary angle.

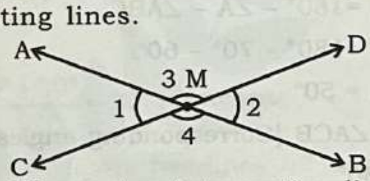
Exp. An angle is more than 45°. What can you say about its complementary angle? Is it more than 45°, equal to 45° or less than 45°?

Idea Let θ_1 and θ_2 two complementary angles

$\therefore \theta_1 + \theta_2 = 90°$

If θ_1 is more than 45° then θ_2 must be less than 45°.

(iii) Vertically opposite angles:→ These are the angles which are equal and opposite to each other at a vertex which is created by two straight intersecting lines.



AB and CD are two intersecting lines, intersect at M.

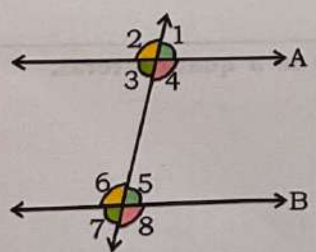
∠1 = ∠2 (vertically opposite angles)

∠3 = ∠4 (vertically opposite angles)

- Each pair of vertically opposite angles are equal.

Angles made by a transversal line

(i) Corresponding angles:→ These are the angles which are formed at corresponding corners when two parallel lines are intersected by transversal line.

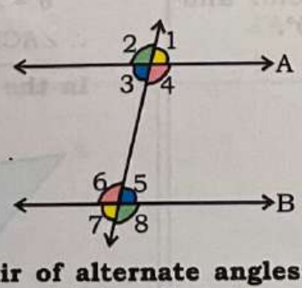


Pair of corresponding angles are

∠1 = ∠5, ∠4 = ∠8

∠2 = ∠6, ∠3 = ∠7

(ii) Alternate angles:→ These are the angles which are formed on opposite sides of the transversal line when two parallel lines are intersected by that transversal line.

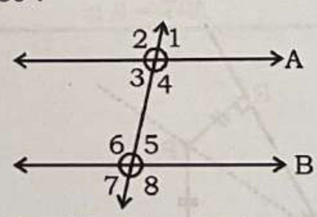


Pair of alternate angles:

Here, interior alternate angles are **∠3 = ∠5, ∠4 = ∠6**

exterior alternate angles are **∠1 = ∠7, ∠2 = ∠8**

(iii) Interior angles:→ Two pairs of interior angles are formed on either side of transversal when two parallel lines are cut by a transversal line. The pairs of interior angles so formed are supplementary i.e., their sum is 180°.

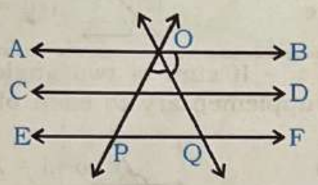


Pair of interior angles:

∠4 + ∠5 = 180°

and **∠3 + ∠6 = 180°**.

Exp. In the figure, the lines AB, CD & EF are parallel lines. $\angle BOQ = 50°$ and OP = OQ, then find the value of $\angle POQ$.



Idea $\angle BOQ = 50°$

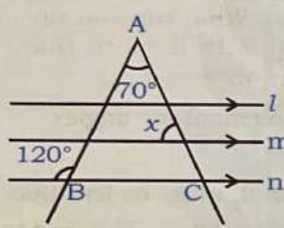
$\therefore \angle OQP = \angle BOQ = 50°$ (Alternate angles)

Since OP = OQ

$\therefore \angle OPQ = \angle OQP = 50°$

$\therefore \angle POQ = 180° - 50° - 50° = $ **80°**

Exp. In the given figure $l \parallel m \parallel n$, then value of x is:



Idea $\angle ABC = 180° - 120° = 60°$

$\Rightarrow \angle ACB = 180° - \angle A - \angle ABC$

$\Rightarrow \angle ACB = 180° - 70° - 60°$

$\Rightarrow \angle ACB = 50°$

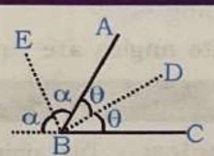
$\Rightarrow \angle x = \angle ACB$ [Corresponding angles]

$\Rightarrow \angle x = 50°$

$\therefore$ The value of x is **50°**.

Angle Bisector

◈



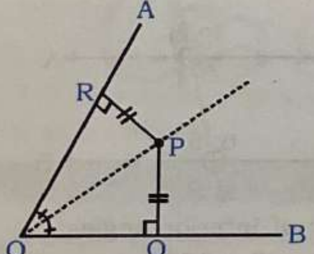
BD is interior angle bisector of $\angle ABC$.

BE $\rightarrow$ exterior angle bisector of $\angle ABC$

$2\alpha + 2\theta = 180°$ **$\alpha + \theta = 90° = \angle EBD$**

$\therefore$ Angle between internal angle bisector and external angle bisector of an angle is 90°.

◈

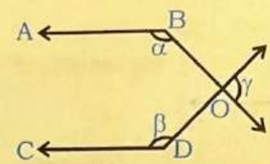


P is any point on angle bisector of $\angle AOB$.

PR = PQ (Result)

Some Useful Results

◈ **If AB ∥ CD then find the value of $\alpha+\beta+\gamma$?**

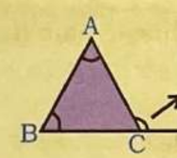


$\alpha + \beta + \gamma = 360°$

◈ **Exterior angle is equal to sum of opposite interior angles.**

$\angle A + \angle B + \angle C = 180°$

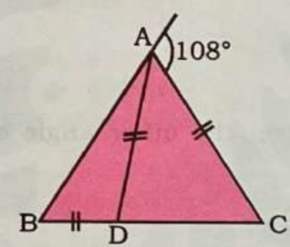
$\angle A + \angle B = 180° - \angle C$



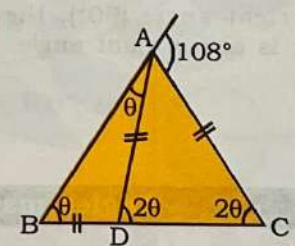
Exterior angle at vertex C = $180° - \angle C = \angle A + \angle B$

sum of all exterior angles = 360°

Exp. In the given triangle, if AD = BD = AC then the value of angle C will be?



Idea



$\angle ADC = \theta + \theta = 2\theta$ (exterior angle)

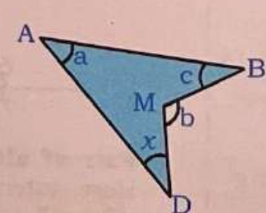
AD = AC $\therefore \angle ADC = \angle ACD = 2\theta$

$\therefore \theta + 2\theta = 108°$ (exterior angle)

$\theta = 36°$

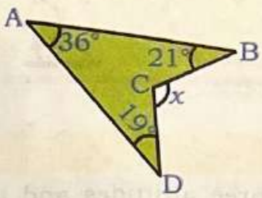
$\therefore \angle ACB = 2\theta = $ **72°**

◈ **In the figure, ABMD is a quadrilateral.**



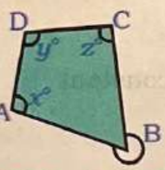
b = a + c + x

Exp. In the given figure, find the value of x.



$x^\circ = 36^\circ + 21^\circ + 19^\circ = 76^\circ$

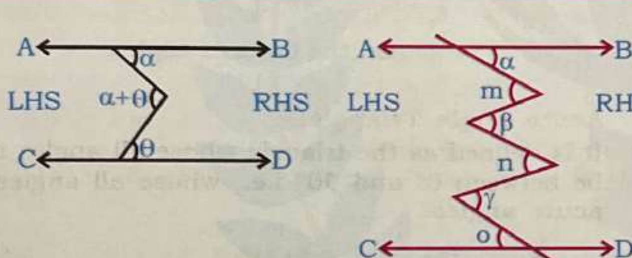
◈ **In the given figure, ABCD is a quadrilateral.**



∠B (internal) = $360^\circ - (x^\circ + y^\circ + z^\circ)$

∠B (external) = $x^\circ + y^\circ + z^\circ$

◈

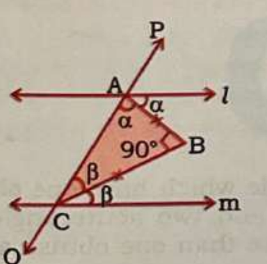


$\alpha+\beta+\gamma = m+n+o$

Sum of angle on RHS = Sum of angle on LHS

$\alpha+\theta = \alpha+\theta$

Exp. In the given figure line l and m are parallel to each other and AB and CB are the angle bisector of ∠A and ∠C respectively then ∠ABC = 90°.



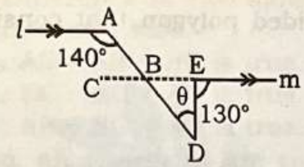
Idea $2\alpha + 2\beta = 180^\circ$ {interior angle of same side}

$\alpha + \beta = 90^\circ$

$\alpha + \beta + \angle ABC = 180^\circ$

$\angle ABC = 90^\circ$

Exp. In the given figure line l and m are parallel to each other then find the value of θ?



Idea $\angle A + \angle ABC = 180^\circ$ {Interior angle of same side}

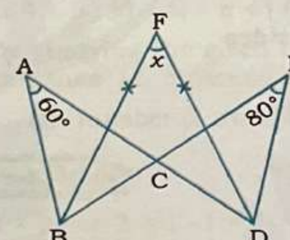
$\angle ABC = 40^\circ = \angle EBD$ (Vertically opposite angle)

$\Rightarrow \angle EBD + \theta = 130^\circ \Rightarrow \theta = 130^\circ - 40^\circ = 90^\circ$

Trick $\theta = (\angle A + \angle E) - 180^\circ = (140^\circ + 130^\circ) - 180^\circ$

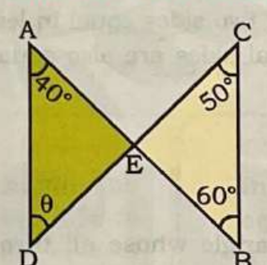
$\theta = 270^\circ - 180^\circ = 90^\circ$

Exp. In the given figure BF and DF are the angle bisector of ∠ABC and ∠EDC respectively then find the value of x?



Idea $x = \dfrac{60^\circ + 80^\circ}{2} = 70^\circ$ **{Concept}**

Exp. In the given figure two lines AB and CD intersect each other at E. Find the value of θ?



Idea $\angle A + \angle D + \angle AED = 180^\circ \Rightarrow \angle C + \angle B + \angle CEB = 180^\circ$

$\angle AED = \angle CEB$ {Vertically opposite angle}

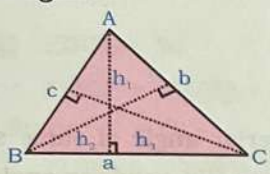
$\angle A + \angle D = \angle C + \angle B$ **{Concept}**

$40^\circ + \theta = 50^\circ + 60^\circ \Rightarrow \theta = 70^\circ$

02 Types of Triangles

Triangle

- A triangle is a three-sided polygon that consists of three edges, three vertices, three altitudes and three angles.



$\angle A + \angle B + \angle C = 180°$

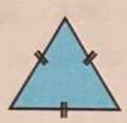
Area $\Rightarrow \frac{1}{2} \times a \times h_1 = \frac{1}{2} bh_2 = \frac{1}{2} ch_3 = \frac{1}{2}$ × Base × Corresponding height. $\Rightarrow ah_1 = bh_2 = ch_3$ = constant

$h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$ **OR** $a : b : c = \frac{1}{h_1} : \frac{1}{h_2} : \frac{1}{h_3}$

Types of triangles

On the basis of sides

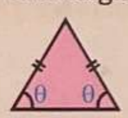
- **Equilateral Triangle**



It has three equal sides and each angle is equal to 60°.

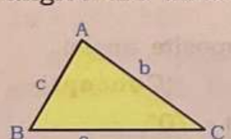
- **Isosceles Triangle:→**

It is a triangle that has any two sides equal in length and angles opposite to equal sides are also equal.



- **Scalene Triangle:→**

It can be defined as a triangle whose all three sides have different lengths and all the three angles are different.

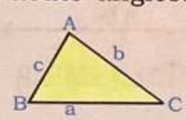


$\angle A \neq \angle B \neq \angle C$ & $a \neq b \neq c$

On the basis of angles

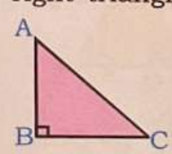
- **Acute Angle Triangle:→**

It is defined as the triangle whose all angles must be between 0° and 90° i.e., whose all angles are acute angles.



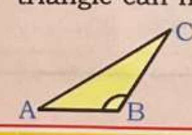
- **Right Angle Triangle:→**

It is a triangle in which two sides are perpendicular, forming a right angle (90°) between them. The side opposite to the right angle is called the hypotenuse. The sides adjacent to the right angle are called legs. The other two angles of the right triangle are acute angles.



- **Obtuse Angle Triangle:→**

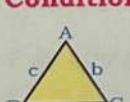
It is defined as the triangle which have one obtuse angle (greater than 90°) and two acute angles. A triangle can not have more than one obtuse angle.



Inequality of triangle

- The triangle inequality states that for any triangle the sum of the length of any two sides must be greater than the length of the third side.
- **Conditions for formation of triangle:→**

1.



$$\boxed{\begin{aligned} |b-c| &< a < b+c \\ |a-c| &< b < a+c \\ |a-b| &< c < a+b \end{aligned}}$$

Exp. For the following sides of triangles, find out if triangle is possible or not?

(i) 4, 9, 15, (ii) 5, 10, 15 (iii) 7, 12, 15

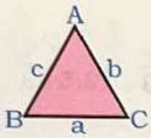
Idea 4, 9, 15 Δ not possible $\because 4 + 9 < 15$

5, 10, 15 Δ not possible $\because 5 + 10 = 15$

7, 12, 15 Δ is possible $\because 7 + 12 > 15$

OR $7+15 > 12$ OR $12 + 15 > 7$ OR $15 - 12 < 7$

2. Sum of any two sides is always greater than 3rd side.



$a+b > c$, $b+c > a$, $c+a > b$

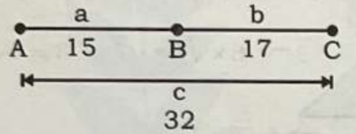
3. Difference of any two sides is always less than third side.

$\Rightarrow b > c - a$ $\Rightarrow b > a - c$

$\Rightarrow \therefore |c - a| < b < c + a$

4. When one side is equal to the sum of other two sides, it does not form a triangle.

If $a + b = c$, then a, b, c are collinear and do not form a triangle.



Exp. If 10, 17, x are sides of a triangle and x is an integer. Find the total number of triangles possible from these sides.

Idea If 10, 17, x are sides of a Δ, $x \to$ integer

Then $17 - 10 < x < 10 + 17 \Rightarrow 7 < x < 27$

$\therefore x \to \{8, 9, 10, ... 26\} \Rightarrow x_{min} = 8, x_{max} = 26$

$x_{total} = 19$ values possible

$\therefore$ **19** Δ's possible

Note Possible values of $x = 2 \times$ small side -1

$\Rightarrow 2 \times 10 - 1 = 19$

Exp. Consider the following inequalities in respect of any triangle ABC:

1. AC – AB < BC
2. BC – AC < AB
3. AB – BC < AC

Which of the above are correct?

Idea Difference of two sides of any triangle is less than third side.

1. AC – AB < BC is true.
2. BC – AC < AB is true.
3. AB – BC < AC is true.

So, all statement are correct.

Exp. If a and b are the lengths of two sides of a triangle such that the product ab = 24, where a and b are integers, then how many such triangles are possible?

Idea It is given that ab = 24, we can have different possible values of a and b such that:

$\Rightarrow 1 \times 24 : (24 - 1) < c < (24 + 1), c \Rightarrow 24$

$\Rightarrow 2 \times 12 : (12 - 2) < c < (12 + 2), c \Rightarrow 11, 12, 13$

$\Rightarrow 3 \times 8 : (8 - 3) < c < (8 + 3), c \Rightarrow 6, 7, 8, 9, 10$

$\Rightarrow 4 \times 6 : (6 - 4) < c < (6 + 4), c \Rightarrow 3, 4, 5, 6, 7, 8, 9$

By studying the given conditions, the third side, c can take 16 different values.

$\therefore$ The number of triangles possible for ab = 24 is **16**.

Alternatively

$1 \times 24 \Rightarrow 2 \times 1 - 1 = 1$

$2 \times 12 \Rightarrow 2 \times 2 - 1 = 3$

$3 \times 8 \Rightarrow 2 \times 3 - 1 = 5$

$4 \times 6 \Rightarrow 2 \times 4 - 1 = 7$

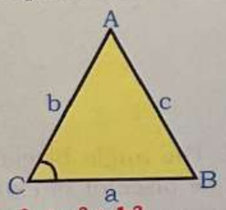
$\therefore 1 + 3 + 5 + 7 =$ **16** triangles possible

Relation between 3 sides of Triangle

- In any triangle the side opposite to the largest angle will be largest and the side opposite to the smallest angle is smallest.

I. Acute Angle Triangle:→

In acute angle triangle the square of the longest side is less than the sum of the squares of two smaller sides.



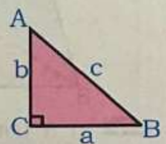
$c^2 < a^2 + b^2$

$\angle C$ = largest angle

side AB = side c = largest side

II. Right Angle Triangle:→

In right angle triangle the square of the longest side is equal to the sum of the squares of two smaller sides.



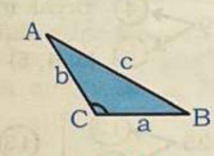
$c^2 = a^2 + b^2$

$\angle C$ = largest angle

side AB = side c = largest side

III. Obtuse Angle Triangle:→

In Obtuse angle triangle the square of the longest side is greater than the sum of the squares of two smaller sides.



$c^2 > a^2 + b^2$

$\angle C$ = largest angle

side AB = side c = largest side

Exp. If sides of triangle are 11.7, 16.9, 23.4. Which type of triangle it is?

Idea Take ratio of sides 11.7 : 16.9 : 23.4

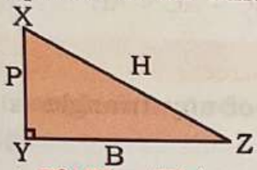
9 : 13 : 18

$18^2 > 9^2 + 13^2$

$\therefore$ Triangle is **scalene and obtuse angle triangle.**

Pythagoras Triplets

- **(Pythagoras theorem):→**
It states that, in a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



$\therefore$ **$P^2+B^2 = H^2$** According to Pythagoras theorem.

- **Pythagorean triplets:** A set of three integers x, y, z is called a Pythagorean triplet if they satisfy the Pythagorean theorem that is, $x^2 + y^2 = z^2$.
- The smallest Pythagorean triplet is (3, 4, 5). It is the only Pythagorean triplet whose sides are in an Arithmetic Progression (AP).
- If x, y, z set is a Pythagorean triplet, then any pair of products or quotients derived from this triplet will also form a triplet.

Hence, $(xk,\ yk,\ zk)$ and $\left(\frac{x}{k}, \frac{y}{k}, \frac{z}{k}\right)$ is also phythagorean triplet.

Some important triplets:→

3 4 5 $\xrightarrow{\times 1/2}$ 1.5, 2, 2.5
3 4 5 $\xrightarrow{\times 2}$ 6, 8, 10
3 4 5 $\xrightarrow{\times 3}$ 9, 12, 15
3 4 5 $\xrightarrow{\times 4}$ 12, 16, 20
3 4 5 $\xrightarrow{\times 5}$ 15, 20, 25
3 4 5 $\xrightarrow{\times \sqrt{2}}$ $3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$

5, 12, 13 $\xrightarrow{\times 2}$ 10, 24, 26
5, 12, 13 $\xrightarrow{\times 3}$ 15, 36, 39

7, 24, 25 $\xrightarrow{\times 1/2}$ 3.5, 12, 12.5
7, 24, 25 $\xrightarrow{\times 2}$ 14, 48, 50
7, 24, 25 $\xrightarrow{\times 3}$ 21, 72, 75

$(2n,\ n^2-1,\ n^2+1)$ $\left(\frac{n^2+1}{2}, \frac{n^2-1}{2}, n\right)$

$(2ab,\ |a^2-b^2|,\ a^2+b^2)$

1, 1, $\sqrt{2}$
1, $\sqrt{3}$, 2
9, 40, 41
11, 60, 61
12, 35, 37
13, 84, 85
16, 63, 65
20, 21, 29
28, 45, 53
33, 56, 65
36, 77, 85
39, 80, 89
48, 55, 73
65, 72, 97
20, 99, 101
15, 112, 113

Finding the triplet

- **Odd number:→**

③ → $3^2 = 9$ → ④, ⑤ Break in two parts at a difference of 1 (3, 4, 5)
↓ no. ↓ $(\text{no.})^2$

⑤ → $5^2 = 25$ → ⑫, ⑬ | ⑬ → 169 → ⑧④, ⑧⑤

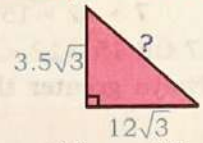
- **Even number:→**

⑧ → $\frac{8^2}{2} = 32$ → ⑮, ⑰ Break in two parts at the diff. of 2. (8, 15, 17)
↓ no. ↓ $\frac{(\text{no.})^2}{2}$

⑥ → $\frac{36}{2} = 18$ → ⑧, ⑩ (6, 8, 10)

Exp. 1: If two sides of a right angle triangle are $3.5\sqrt{3}$ and $12\sqrt{3}$. Find the third side.

Idea

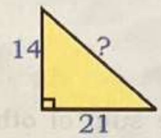


$7 \xrightarrow{\div 2} 3.5$ $\quad 24 \xrightarrow{\div 2} 12$ $\quad 25 \xrightarrow{\div 2} 12.5 \Rightarrow 3.5\sqrt{3},\ 12\sqrt{3},\ 12.5\sqrt{3}$

$\therefore$ 3rd side = $\mathbf{12.5\sqrt{3}}$

Exp. 2: If two sides of a right angle triangle are 14 and 21. Find the third side?

Idea

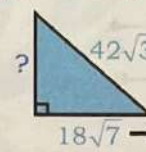


$\sqrt{3^2+2^2} = \sqrt{13}$

⑦×2 ⑦×3 ⑦$\times\sqrt{13}$ = $\mathbf{7\sqrt{13}}$

Exp. 3: If two sides of a right angle triangle are $42\sqrt{3}$ (hypotenuse) and $18\sqrt{7}$. Find the third side?

Idea



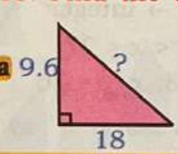
$42\sqrt{3} \rightarrow 6\times\sqrt{7}\times\sqrt{7}\times\sqrt{3} = 6\sqrt{7}\times\sqrt{21}$

$18\sqrt{7} \rightarrow 6\sqrt{7}\times 3$

3rd side = $6\sqrt{7}\times\sqrt{21-9} = 6\sqrt{7}\times\sqrt{12}$ = $\mathbf{12\sqrt{21}}$

Exp. 4: If two sides of a right angle triangle are 9.6 and 18. Find the third side.

Idea



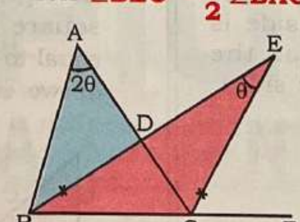
9.6 : 18
×1.2
8 : 15 : 17 $\xrightarrow{\times 1.2}$ **20.4**

Some properties of triangle

Property : 1

- In triangle ABC, the angle bisector of internal angle $\angle B$ and angle bisector of external angle $\angle C$ meets at E.

Then $\angle BEC = \frac{1}{2}\angle BAC$



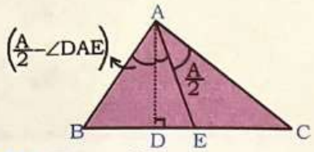
Exp. In a triangle ABC, $\angle A = 70°$. The angle bisector of internal angle $\angle B$ and the angle bisector of external angle $\angle C$ will meet at E. Find the value of $\angle BEC$.

Idea $\angle BEC = \frac{\angle BAC}{2} = \frac{70°}{2} = \mathbf{35°}$

Property : 2

- In a triangle ABC, AE is the angle bisector of $\angle BAC$ and AD is the perpendicular drawn on side BC.

Then $\angle DAE = \frac{\angle B - \angle C}{2}$



Proof: AD → Altitude

AE → Angle bisector of ∠A

$\angle B + \frac{\angle A}{2} - \angle DAE = 90°$ [External angle property]

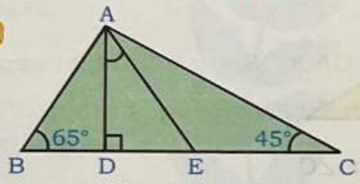
$\Rightarrow \angle DAE = \frac{\angle A}{2} - 90° + \angle B$

$= \frac{\angle A}{2} - \frac{\angle A}{2} - \frac{\angle B}{2} - \frac{\angle C}{2} + \angle B \quad \because \left(\frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2} = 90°\right)$

$\Rightarrow \angle DAE = \frac{\angle B}{2} - \frac{\angle C}{2} = \frac{\angle B - \angle C}{2}$

Exp. In a triangle ABC, AE is the angle bisector of ∠BAC and AD is the perpendicular drawn on side BC. If ∠ABC is 65° and ∠ACB is 45°, then find ∠DAE.

Idea



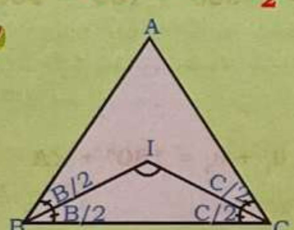
$\angle DAE = \frac{65° - 45°}{2} \Rightarrow \frac{20°}{2} = 10°$

Property : 3

◈ In a triangle ABC, the bisector of ∠B and ∠C meet at I.

Then $\angle BIC = 90° + \frac{\angle A}{2}$

Proof:



$\angle A + \angle B + \angle C = 180°$

$\angle A = 180° - (\angle B + \angle C) \Rightarrow \frac{\angle B + \angle C}{2} = \frac{180° - \angle A}{2}$

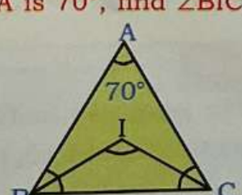
$\angle BIC = 180° - \left(\frac{\angle B + \angle C}{2}\right)$

$= 180° - \left(\frac{180° - \angle A}{2}\right) = 90° + \frac{\angle A}{2}$

$\therefore \angle BIC = 90° + \frac{\angle A}{2}$

Exp. In triangle ABC bisector of ∠B and ∠C meet at I. If ∠A is 70°, find ∠BIC.

Idea

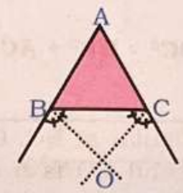


$\angle BIC = 90° + \frac{\angle A}{2} \Rightarrow 90° + \frac{70°}{2} \Rightarrow 125°$

Property : 4

◈ In a triangle ABC, the bisectors of external angle ∠B and ∠C meets at O.

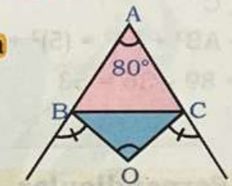
Then



$\angle BOC = 90° - \frac{\angle A}{2} \qquad \angle A = 2(90° - \angle BOC)$

Exp. In triangle ABC, the angle bisectors of external angle ∠B and ∠C meets at O. If ∠A is 80°. Find ∠BOC?

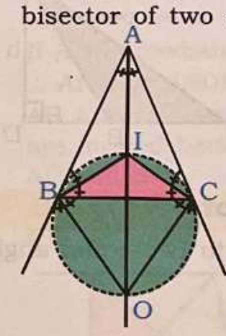
Idea



$\angle BOC = 90° - \frac{\angle A}{2} \Rightarrow 90° - \frac{80°}{2} \Rightarrow 50°$

Property : 5

◈ When interior angle bisector and exterior angle bisector of two angles of a triangle are given.



$\angle BIC = 90° + \frac{\angle A}{2} \qquad \angle BOC = 90° - \frac{\angle A}{2}$

AIO will be a straight line and bisect angle A.

$\angle BIC + \angle BOC = 180°$

BICO will be a cyclic quadrilateral.

Property : 6

◈ In a triangle ABC, the side BC produced to D and angle bisector of ∠A meets BC at E. Then

$\angle ABC + \angle ACD = 2\angle AEC$

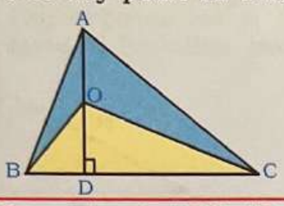
Exp. Bisector of ∠A meets BC at E. If ∠ABC = 40° and ∠AEC = 70°. Then find, ∠ACD.

Idea $\angle ABC + \angle ACD = 2\angle AEC \Rightarrow 40° + \angle ACD = 2 \times 70°$

$\angle ACD = 140° - 40° = 100°$

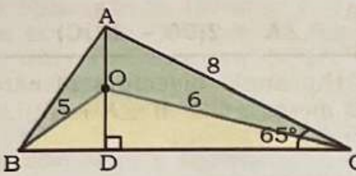
Property : 7

- In any ΔABC, $AD \perp BC$.
  O is any point on altitude



$AB^2 + OC^2 = OB^2 + AC^2$

Exp. In a triangle ABC, AD is perpendicular at BC. O is any point on AD. If BO is 5 cm and CO is 6 cm. Find the length of AB if length of AC is 8 cm.



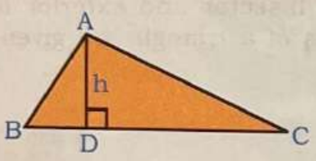
Idea $AB^2 + OC^2 = OB^2 + AC^2 \Rightarrow AB^2 + (6)^2 = (5)^2 + (8)^2$

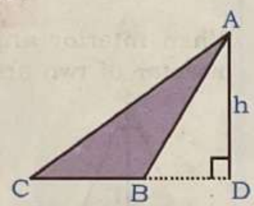
$AB^2 + 36 = 25 + 64 \Rightarrow AB^2 = 89 - 36 = 53$

$AB = \sqrt{53}$ **cm**

Altitude / Height / Perpendicular

- The perpendicular drawn from the vertex of the triangle to the opposite side.

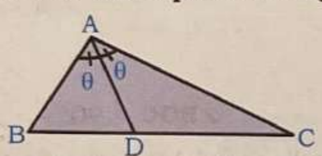




Here, AD is perpendicular.

Angle bisector

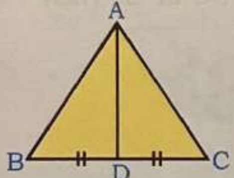
- A line that splits an angle into two equal angles.



AD is the angle bisector of $\angle BAC$ and BD and DC need not be equal

Median

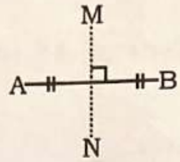
- Line drawn from a vertex to opposite side which divides the opposite side into equal parts.



AD is the median of side BC. **(BD = DC)**

Perpendicular bisector

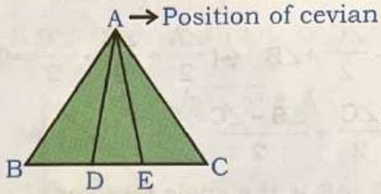
- A perpendicular bisector is a line that bisects a line segment in two equal parts and makes an angle of 90° at the point of intersection.



MN is the perpendicular bisector of line segment AB.

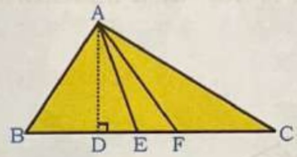
Cevian

- **Cevian:→** Any line which joins vertex to opposite side.



AD, AE are cevians

- Δ ABC is scalene Δ



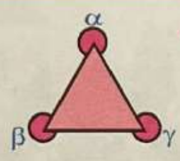
$AC > AB \quad \therefore \angle B > \angle C$

$\perp$ AD will be near to largest among $\angle B$ and $\angle C$ i.e angle $\angle B$ and far from small angle $\angle C$.

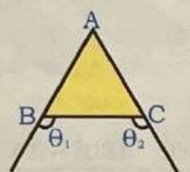
AE → Angle bisector of $\angle A$

AF → median i.e. **BF = FC**

Some other Results

- 

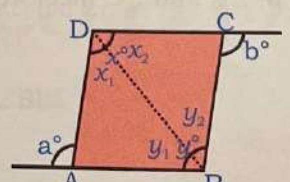
  $\alpha+\beta+\gamma = 3\times360° - 180° = 900°$

- 

  $\theta_1 + \theta_2 = 180° + \angle A$

- If angles of a Δ are in A.P., middle angle is always 60°.
  Let three angles are:→
  (a–d), a, (a+d)
  $\therefore$ a–d+a+a+d = 180° 3a = 180° a = 60°
  $\therefore \angle A + \angle C = 120°$ & $\angle B = 60°$

$\angle A$	$\angle B$	$\angle C$
↓	↓	↓
$60°^-$	60°	$60°^+$

- 

  External angle property
  $x_1 + y_1 = a°$ $x_2 + y_2 = b°$
  $x_1 + x_2 + y_1 + y_2 = a+b$ $\boldsymbol{x + y = a + b}$

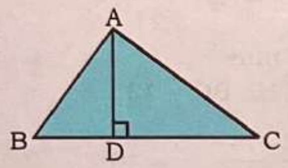
03 Area (Side Properties)

Area of triangle

- To find the area of triangle we can use following methods.

Method → 1

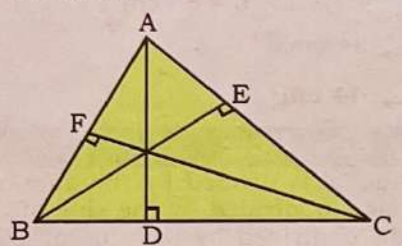
- **When height of the triangle is known.**



Area of $\Delta ABC = \frac{1}{2} \times$ Base $\times$ perpendicular on that base.

Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

If we draw perpendicular to all sides of the triangle.



To calculate area of ΔABC, take any side as base and the perpendicular drawn on that side will be height.

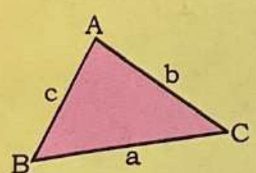
Area of $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times AB \times CF = \frac{1}{2} \times AC \times BE$

Method → 2

- When height of the triangle is not known.

Semi-perimeter (s) $= \frac{a+b+c}{2}$

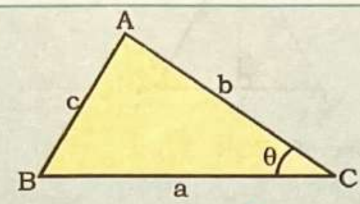
Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$



Method → 3

- When two sides of the triangle is known and the angle between them is given.

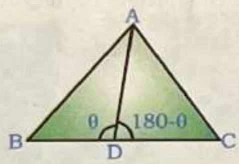
Area of $\Delta ABC = \frac{1}{2}ab \sin\theta$



$\therefore$ **Area of $\Delta ABC = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B = \frac{1}{2}bc \sin A$**

Some important point related to area

1. In a ΔABC, a cevian AD is drawn meeting side BC at D. Then ratio of area of ΔADB to area of ΔADC is BD : DC.



$$\frac{\text{Area of } \Delta ADB}{\text{Area of } \Delta ADC} = \frac{\frac{1}{2} \times AD \times BD \times \sin\theta}{\frac{1}{2} \times AD \times DC \times \sin(180-\theta)} = \left(\frac{BD}{DC}\right)$$

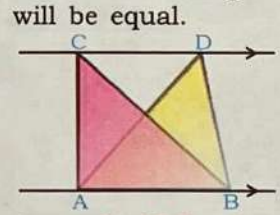
$\therefore$ **The ratio which will divide cevian base, area will also be divided in the same ratio.**

If AD is median BD = DC

$\therefore$ Area of ΔADB = Area of ΔADC

2. AB and CD are two parallel lines. Two triangles are formed between these parallel lines by joining AD and BC.

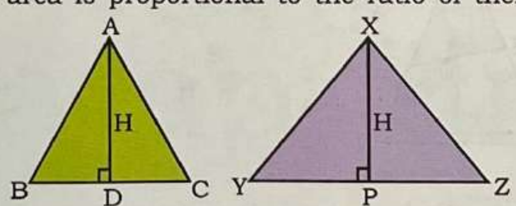
Area of two triangles ΔABC and ΔABD thus formed will be equal.



Area of ΔABC = Area of ΔABD; AB $\parallel$ CD

Note If AB || CD, area of triangles formed between the parallel line and on same base are equal.

3. If height of two triangle is same, the ratio of their area is proportional to the ratio of their bases.

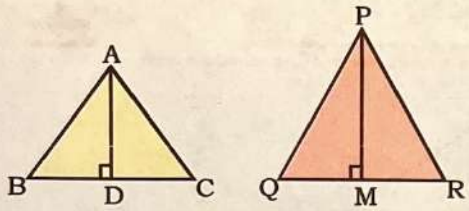


Height of ΔABC = Height of ΔXYZ

$\therefore$ AD = XP = H

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta XYZ} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times YZ \times XP} = \frac{BC \times H}{YZ \times H} = \frac{BC}{YZ}$$

4. If the base of two triangles is same, the ratio of their areas is proportional to the ratio of their heights.



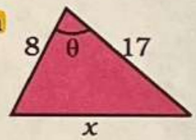
Base of ΔABC = Base of ΔPQR

$\therefore$ BC = QR

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PM} = \frac{\mathbf{AD}}{\mathbf{PM}}$$

Exp. If three sides of a triangle are 8cm, 17 cm and x cm. for what value of x when area of triangle is maximum?

Idea



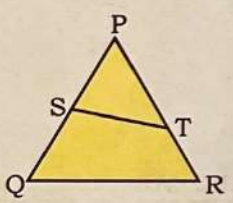
Area of $\Delta = \frac{1}{2} \times 8 \times 17 \times \sin\theta$ $\qquad \Delta_{max} = \sin\theta_{max}$

Area of $\Delta_{max} = \frac{1}{2} \times 8 \times 17 \times 1 = 68\text{cm}^2$ $\qquad \sin\theta_{max} = 1$

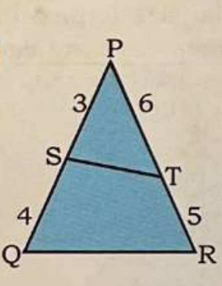
at $\theta = 90°$

$x = \sqrt{8^2 + 17^2} = \sqrt{353} = \mathbf{18.79 cm}$

Exp. In the given triangle PQR, S and T are two points on side PQ and PR respectively such that PS:SQ = 3:4 and PT:TR = 6:5, if area of □STQR = 177 cm² then find area of ΔPQR?



Idea

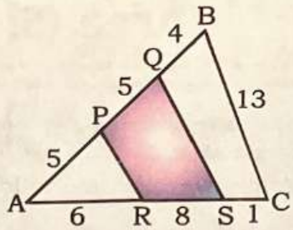


$$\frac{Ar\Delta PST}{Ar\Delta PQR} = \frac{3 \times 6 \times \sin P}{7 \times 11 \times \sin P} = \frac{3 \times 6}{7 \times 11} = \frac{18}{77}$$

Ar □STRQ = 77–18 = 59 unit $\xrightarrow{\times 3}$ 177 cm²

$\therefore$ Ar ΔPQR = **77 × 3 = 231 cm²**

Exp. Find the shaded area?



Idea Ar ΔAPR : ΔAQS : ΔABC

5 × 6 : 10 × 14 : 14 × 15

3 : 14 : 21

Ar □ PQSR = 14–3 = 11 unit

Here, AB = 14cm, AC = 15, BC = 13

$$\text{Semiperimeter (s)} = \frac{13+14+15}{2} = 21$$

$$\therefore \text{Area of } \Delta ABC = \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6}$$

$$= \sqrt{7 \times 3 \times 4 \times 2 \times 7 \times 6}$$

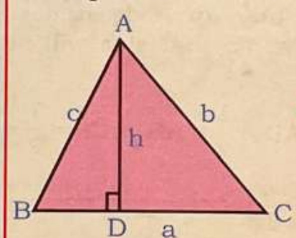
$$= 7 \times 2 \times 6 = 84 \text{ cm}^2$$

$\therefore$ 21 unit $\xrightarrow{\times 4}$ 84 cm²

11 unit $\xrightarrow{\times 4}$ **44 cm²**

Sine Rule

- In a triangle, side 'a' is divided by the sine of $\angle A$ is equal to the side 'b' divided by the sine of $\angle B$ is equal to the side 'c' divided by the sine of $\angle C$.



$$\frac{\mathbf{a}}{\mathbf{sinA}} = \frac{\mathbf{b}}{\mathbf{sinB}} = \frac{\mathbf{c}}{\mathbf{sinC}} = \mathbf{K\ (constant)}$$

a:b:c = KsinA : KsinB : KsinC

a:b:c = sinA : sinB : sinC

Area of $\Delta ABC = \frac{1}{2} \times$ base × height

Area of triangle $= \frac{1}{2} \times a \times h \Rightarrow \frac{1}{2} \times a \times c\sin B$

$\sin B = \frac{h}{c} \Rightarrow h = c\sin B$

$\sin C = \frac{h}{b} \Rightarrow h = b\sin C$

$c\sin B = b\sin C \Rightarrow \frac{c}{\sin c} = \frac{b}{\sin b}$

$\therefore$ **Area of triangle ABC** $= \frac{1}{2}$ **ac sinB** $= \frac{1}{2}$ **ab sinC** $= \frac{1}{2}$ **bc sinA**