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2026
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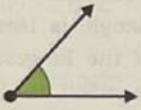


Lines & Angles

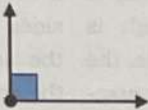
Angle: The inclination between two lines is called an angle. A figure formed by two rays or lines that share a common endpoint is called an angle. The two rays are known as the arms of the angle, and the common endpoint is called the vertex.

Types of angles

1
Acute angle



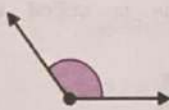
2
Right angle



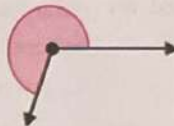
3
Straight angle



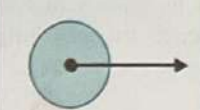
4
Obtuse angle



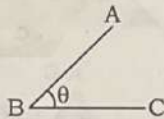
5
Reflex angle



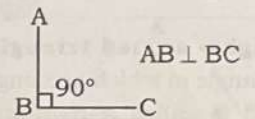
6
Complete angle



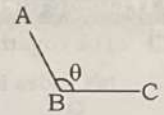
Acute angle: If an angle is greater than 0° and less than 90°
 $0^\circ < \theta < 90^\circ$ ($\angle ABC$ is an acute angle)



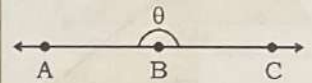
Right angle: If an angle is equal to 90° , it is called a right angle.
 $\theta = 90^\circ$ ($\angle ABC$ is a right angle.)



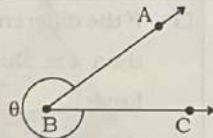
Obtuse angle: If an angle is greater than 90° and less than 180° , it is called an obtuse angle.
 $90^\circ < \theta < 180^\circ$ ($\angle ABC$ is an obtuse angle.)



Straight angle: If an angle is equal to 180° , it is called a straight angle.
 $\theta = 180^\circ$ ($\angle ABC$ is a straight angle)



Reflex angle: If an angle is greater than 180° and less than 360° , it is called a reflex angle.
 $180^\circ < \theta < 360^\circ$ ($\angle ABC$ is a reflex angle)



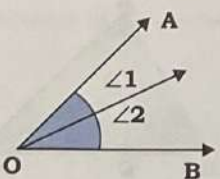
Complete angle: If an angle is equal to 360° , it is called a complete angle.



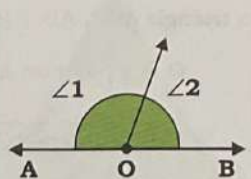
Pair angle

Pair Angles

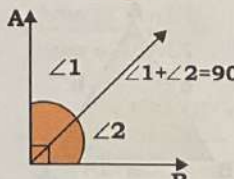
Adjacent Angle



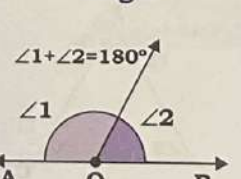
Linear Pair Angle



Complementary Angle



Supplementary Angle



Triangle

Types of Triangles

Based on Angles

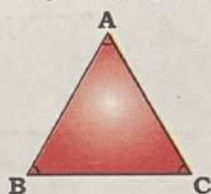
- ❖ Acute-angled Triangle
- ❖ Right-angled Triangle
- ❖ Obtuse-angled Triangle

Based on Sides

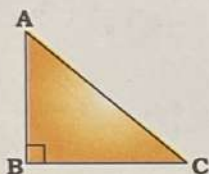
- ❖ Scalene Triangle
- ❖ Isosceles Triangle
- ❖ Equilateral Triangle

Triangle based on angle

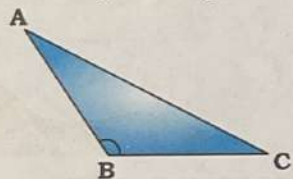
Acute-angled triangle: A triangle in which each angle is less than 90° is called an acute-angled triangle.



Right-angled triangle: A triangle in which one angle is 90° is called a right-angled triangle.

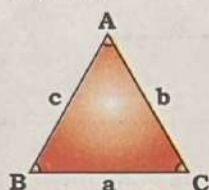


Obtuse-angled triangle: A triangle in which one angle is greater than 90° is called an obtuse-angled triangle.

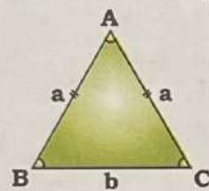


Triangle based on side

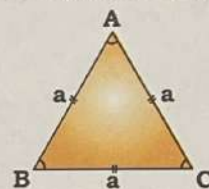
Scalene triangle: A triangle in which all three sides are of different lengths is called a scalene triangle.



Isosceles triangle: A triangle whose two sides are equal is called an isosceles triangle.



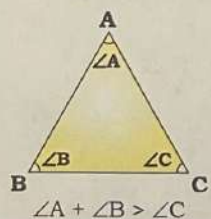
Equilateral triangle: A triangle in which all sides are equal is called an equilateral triangle.



Identifying triangles based on angles and side ratios

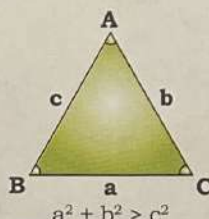
Based on angle ratio

If the sum of the ratios of two angles of a triangle is greater than the third angle, the triangle is called an acute-angled triangle.

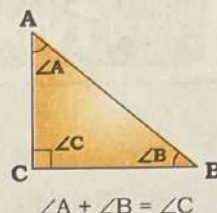


Based on arm ratio

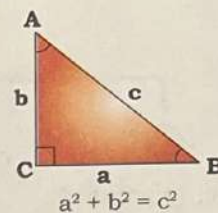
If the sum of the squares of two sides of a triangle is greater than the square of the longest side, the triangle is called an acute-angled triangle.



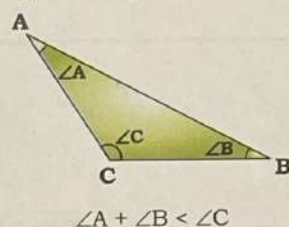
If the sum of the ratios of two angles is equal to the third angle, then the triangle is a right-angled triangle.



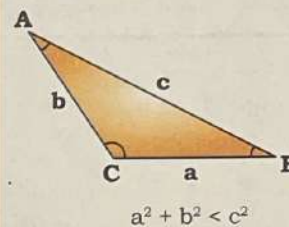
If the sum of the squares of two sides of a triangle is equal to the square of the longest side, the triangle is called a right-angled triangle.



If the sum of the ratios of two angles of a triangle is less than the third angle, the triangle is called an obtuse-angled triangle.



If the sum of the squares of two sides of a triangle is less than the square of the longest side, the triangle is called an obtuse-angled triangle.



Triangle inequality

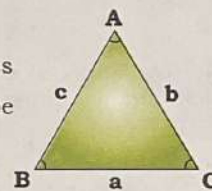
- ❑ The construction of a triangle is possible only if the sum of two sides is greater than the third side.

$$a + b > c, b + c > a, c + a > b$$

- ❑ If the difference between two sides is less than the third side, a triangle will be formed.

$$a - b < c, b - c < a, c - a < b$$

$$\therefore |c - a| < b < (c + a)$$



- ❑ The length of the longest side of a triangle is less than half of its perimeter.

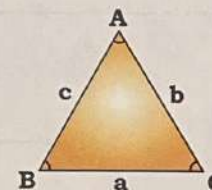
$$c < (a + b)$$

$$(c + c) < (a + b + c)$$

$$2c < \text{perimeter}$$

$$c < \frac{\text{Perimeter}}{2}$$

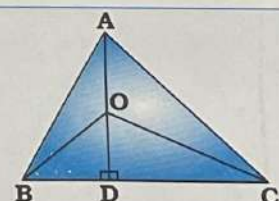
Where c = the length of the longer side.



In a triangle ABC, $AD \perp BC$,

O is a point on AD, then

$$AB^2 + OC^2 = AC^2 + OB^2$$



Based on the angle bisector of a triangle

- In triangle ABC, AE is the angle bisector of $\angle BAC$ and AD is the perpendicular drawn to side BC.

Then

$$\angle DAE = \frac{\angle B - \angle C}{2}$$

- In triangle ABC, the angle bisector of interior angle $\angle B$ and the angle bisector of exterior angle $\angle C$ meet at E.

then, $\angle BEC = \frac{1}{2} \angle BAC$

- In triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet at I.

then, $\angle BIC = 90^\circ + \frac{\angle A}{2}$

- In triangle ABC, the bisectors of the exterior angles at $\angle B$ and $\angle C$ meet at O.

Then,

$$\angle BOC = 90^\circ - \frac{\angle A}{2}$$

$$\angle A = 2(90^\circ - \angle BOC)$$

- The line connecting the points formed by the intersection of two internal angle bisectors and two external angle bisectors of a triangle is always a straight line.

❖ AIO is a straight line and divides A into two equal parts.

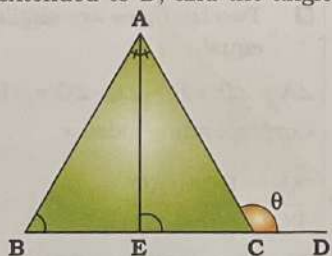
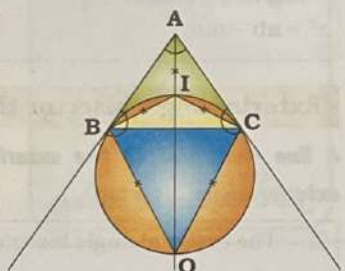
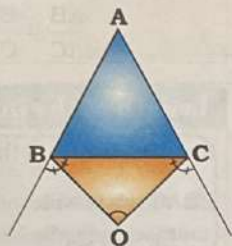
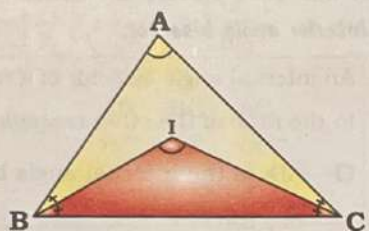
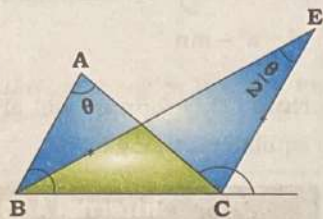
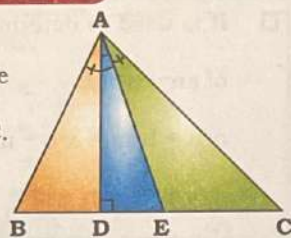
$$\angle BIC + \angle BOC = 180^\circ$$

❖ BICO is a cyclic quadrilateral.

- In a triangle ABC, side BC is extended to D, and the angle bisector of $\angle A$ meets BC at E.

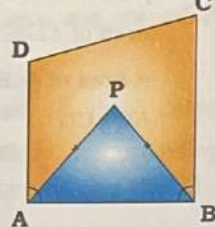
Then

$$\angle ABC + \angle ACD = 2 \angle AEC$$



- In any quadrilateral, the angle formed by the bisectors of two adjacent angles is equal to half the sum of the other two angles.

$$\angle APB = \frac{\angle D + \angle C}{2}$$



- In isosceles triangle

❖ $AB = AC$

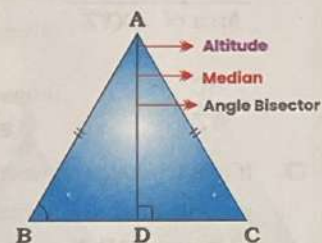
❖ $\triangle ADB \cong \triangle ADC$

❖ If $AD \perp BC \Rightarrow BD = DC$

❖ $AD \Rightarrow$ Altitude to BC,

$\Rightarrow$ Median to BC

$\Rightarrow$ Angle bisector of $\angle A$



Property of area of a triangle

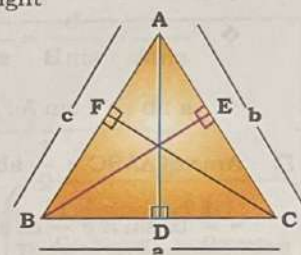
When the height of the triangle is given

- Area of $\triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

- If we draw perpendiculars to all the sides of a triangle.

$$\text{Area of } \triangle ABC = \frac{1}{2} \times a \times h_1 = \frac{1}{2} \times$$

$$b \times h_2 = \frac{1}{2} \times c \times h_3$$



When height is not given in a triangle

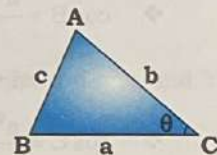
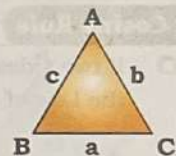
- Area of $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

Where, semi-perimeter $(s) = \frac{a+b+c}{2}$

- When two sides of a triangle are known and the angle between them is given.

$$\text{Area of } \triangle ABC = \frac{1}{2} ab \sin C$$

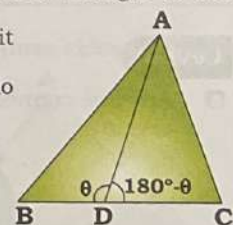
$$= \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$$



- When a line is drawn from a vertex of a triangle to the opposite side, the ratio in which it divides the side is the same as the ratio in which it divides the area.

$$\frac{\text{Area of } \triangle ADB}{\text{Area of } \triangle ADC}$$

$$= \frac{\frac{1}{2} \times AD \times BD \times \sin \theta}{\frac{1}{2} \times AD \times DC \times \sin(180^\circ - \theta)} = \left(\frac{BD}{CD} \right)$$

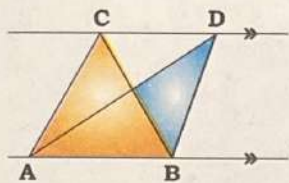


- All triangles formed between the same base and two parallel lines have equal areas.

Area of $\triangle ABC$

= Area of $\triangle ABD$

$AB \parallel CD$

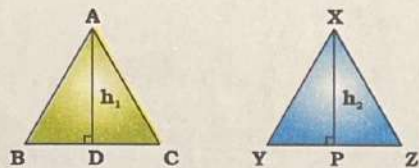


- If two triangles have the same height, the ratio of their areas is proportional to the ratio of their bases.

Area of $\triangle ABC$

Area of $\triangle XYZ$

$$= \frac{BC}{YZ}$$



- If two triangles have equal bases, the ratio of their areas is proportional to the ratio of their heights.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle XYZ} = \frac{h_1}{h_2} = \frac{AD}{XP}$$

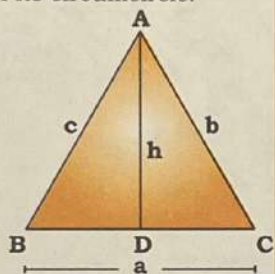
Sine Rule

- The ratio of a side of a triangle to the sine of the opposite angle is equal to twice the radius of its circumcircle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B \end{aligned}$$



Cosine Rule

- If two sides and the angle between them are given, we use the Law of Cosine to find the third side.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

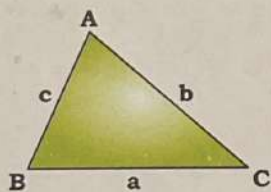
$$\Rightarrow a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow b^2 = a^2 + c^2 - 2ac \cos B$$

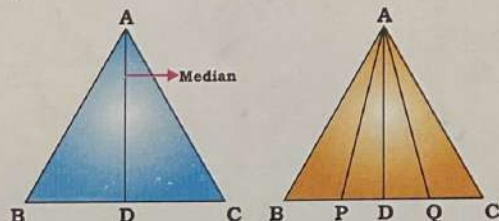
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow c^2 = a^2 + b^2 - 2ab \cos C$$



Cevian

- Any line connecting a vertex to the opposite side is called a cevian.



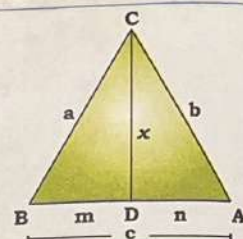
AP, AD, and AQ are cevians.

Stewarts Theorem

- It is used to determine the length of any cevian.

$$a^2n + b^2m = x^2c + mnc$$

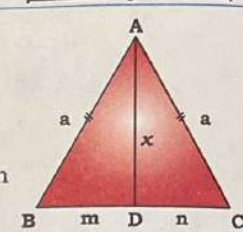
$$\Rightarrow a^2n + b^2m = c(x^2 + mn)$$



In an isosceles triangle ($AB = AC$)

$$x^2 = a^2 - mn$$

Note: This formula will also apply to an equilateral triangle



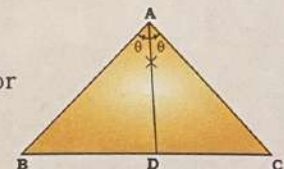
Internal Angle Bisector Theorem

A line that bisects an interior angle of a triangle is called an internal angle bisector.

An internal angle bisector of a triangle divides the opposite side in the ratio of the other two sides.

- AD, is the internal angle bisector of $\angle BAC$

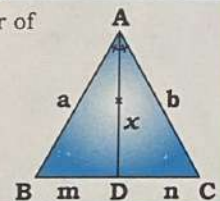
$$\therefore \frac{AB}{AC} = \frac{BD}{CD}$$



Length of the internal angle bisector

In $\triangle ABC$ if AD, is the internal angle bisector of $\angle BAC$ what will be the length of AD?

$$x^2 = ab - mn$$



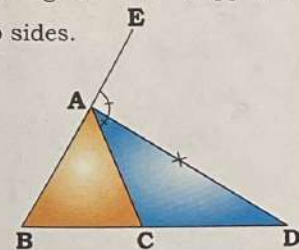
Exterior angle bisector theorem

A line that bisects the exterior angle of a triangle is called an exterior angle bisector.

- The external angle bisector of a triangle divides the opposite side in the ratio of the other two sides.

- AD, is the external bisector of $\angle EAC$

$$\therefore \frac{AB}{AC} = \frac{BD}{CD}$$



Similarity of triangles

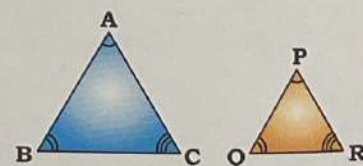
- Two triangles are similar if their corresponding angles are equal.

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

Corresponding sides $\propto$.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

Therefore, $\triangle ABC \sim \triangle PQR$



Properties of symmetry of a triangle

Corresponding lengths are in equal ratio.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{h_1}{h_2} = \frac{r_1}{r_2} = \frac{R_1}{R_2} = \frac{P_1}{P_2} = \frac{s_1}{s_2} = \frac{m_1}{m_2}$$

- h = height, r = inradius,

R = circumradius

- P = perimeter, s

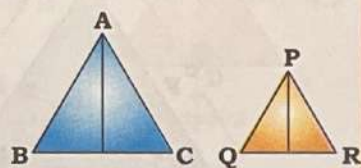
= semiperimeter, m = median

- The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

$$= \left(\frac{h_1}{h_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{P_1}{P_2}\right)^2 = \left(\frac{s_1}{s_2}\right)^2 = \left(\frac{m_1}{m_2}\right)^2$$

- $\sqrt{\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle PQR}} = \text{Side Ratio} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$



Congruency of Triangles

- Triangles having the same shape and the same size are called congruent triangles.
- Two triangles are said to be congruent if their three corresponding sides are equal and their three corresponding angles are equal.

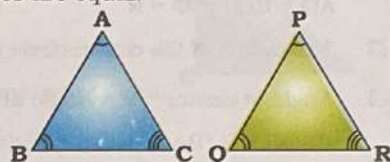
Properties of Congruent Triangles

Two triangles are said to be congruent if:

- Their corresponding angles are equal:

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

$$= \angle R$$



- Their corresponding sides are also equal:

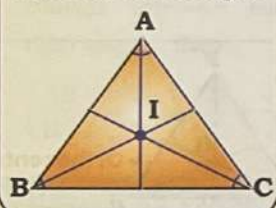
$$AB = PQ, BC = QR, AC = PR$$

Therefore, $\triangle ABC \cong \triangle PQR$

Centre of Triangles

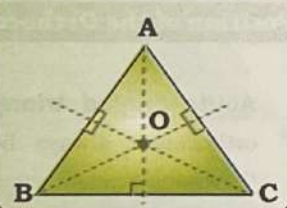
Incenter (I):

The point of intersection of the internal angle bisectors of a triangle.



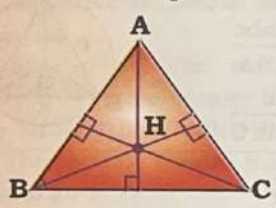
Circumcenter (O):

The point of intersection of the perpendicular bisectors of the sides of a triangle.



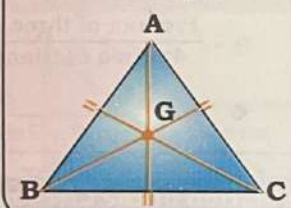
Orthocenter (H):

The point of intersection of the three altitudes of a triangle.



Centroid (G):

The point of intersection of the three medians of a triangle.



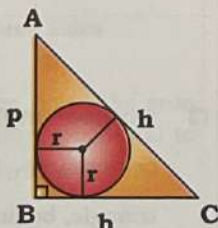
Inradius

- Inradius of any triangle

$$r = \frac{\Delta}{s} = \frac{\text{Area}}{\text{Semi-perimeter}}$$

- Inradius of a right-angled triangle

$$r = \frac{p + b - h}{2}$$



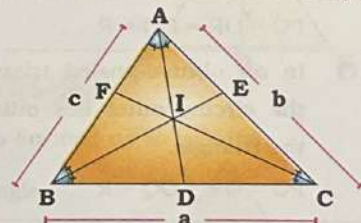
- If h_1 , h_2 and h_3 are the three altitudes of a triangle and 'r' is the inradius, then:

$$\frac{1}{r} = \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3}$$

- Relation between the incenter and the three sides of a triangle

$$\frac{AI}{ID} = \frac{b+c}{a}, \frac{BI}{IE} = \frac{a+c}{b},$$

$$\frac{CI}{IF} = \frac{a+b}{c}$$



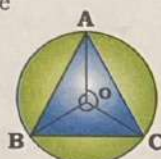
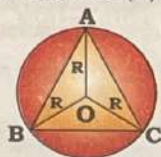
Circumcentre

The point of intersection of the perpendicular bisectors of the three sides of a triangle is called the circumcenter.

- Circumcircle:** The circle passing through all three vertices of a triangle is called the circumcircle.

Properties of the Circumcenter

- The center of the circumcircle is called the circumcenter (O).
- The circumcenter of a triangle is equidistant from all three vertices, and this distance is equal to the circumradius (R)
 $AO = BO = CO = R$
- The radius of the circumcircle is called the circumradius (R).
- Angle at center = $2 \times$ Angle at circumference
 $\angle BOC = 2\angle BAC$,
 $\angle AOC = 2\angle ABC$,
 $\angle AOB = 2\angle ACB$
- The circumcenter may lie inside, outside, or on a side of the triangle.
- In an acute-angled triangle, the circumcenter lies inside the triangle.
- In an obtuse-angled triangle, the circumcenter lies outside the triangle.
- In a right-angled triangle, the circumcenter lies at the midpoint of the hypotenuse.

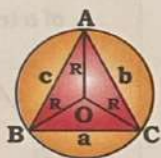


Circumradius

- Circumradius of any triangle

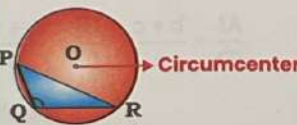
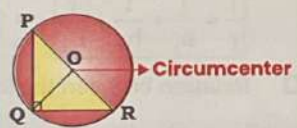
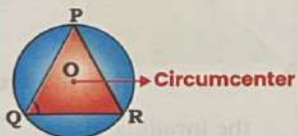
$$R = \frac{\text{Product of three sides}}{4 \times \text{Area of triangle}} = \frac{abc}{4\Delta}$$

$$R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$$



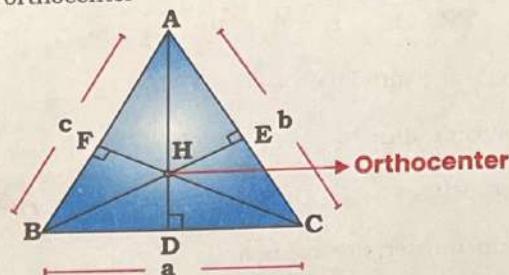
Position of the Circumcenter

- In an acute-angled triangle, the circumcenter always lies inside the triangle.
 $PO = RO = QO = R$
- In a right-angled triangle, the circumcenter lies at the midpoint of the hypotenuse.
 $PO = OR = OQ = R$
- In an obtuse-angled triangle, the circumcenter lies outside the triangle.
 $PO = OR = OQ = R$
- Distance (d) between the incenter and circumcenter of any triangle
 $d = \sqrt{R^2 - 2Rr}$ Where, R = Circumradius and r = Inradius.



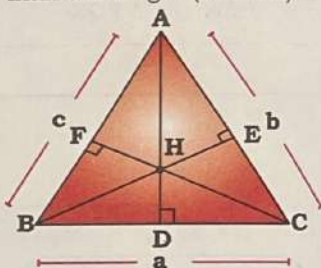
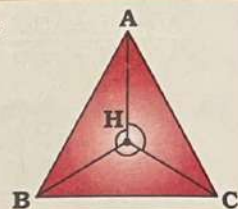
Orthocentre

The point of intersection of the three altitudes of a triangle is called the orthocenter



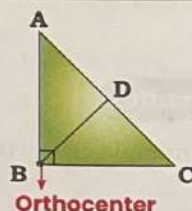
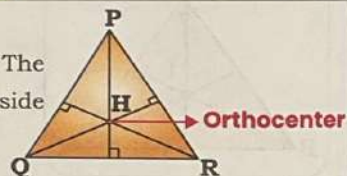
Properties of the Orthocenter

- If H is the orthocenter of $\triangle ABC$, then:
 Angle formed at the orthocenter
 $\angle BHC = 180 - \angle A$
 $\angle AHC = 180 - \angle B$
 $\angle AHB = 180 - \angle C$
- The length of the side is greater than the height (altitude)
 $AB > AD, BC > BE, AC > CF$
- The sum of the three sides of a triangle is greater than the sum of its three altitudes
 $(AB + BC + AC) > (AD + BE + CF)$
- Relation between the altitude and side of a triangle
 $a : b : c = \frac{1}{h_1} : \frac{1}{h_2} : \frac{1}{h_3}$ or $h_1 : h_2 : h_3 = \frac{1}{a} : \frac{1}{b} : \frac{1}{c}$



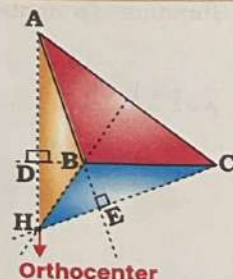
Position of the Orthocenter

- Acute-angled triangle:** The orthocenter always lies inside the triangle.
- Right-angled triangle:** In a right-angled triangle, the orthocenter lies at the vertex of the right angle.



Note Distance between the orthocenter and circumcenter of a right-angled triangle. = $\frac{\text{Hypotenuse}}{2} = \frac{h}{2}$

- Obtuse-angled triangle:** The orthocenter lies outside the triangle, behind the obtuse angle.



Centroid

- Median** A line segment joining a vertex of a triangle to the midpoint of the opposite side is called a median.

❖ A median divides the triangle into two equal-area triangles.

- Centroid:** The point of intersection of the three medians of a triangle is called the centroid.

It is denoted by G.

Where AD, BE, and CF are the medians of the triangle, and G is the centroid

Apollonius' Theorem

- Apollonius' Theorem is used to find the length of the medians of a triangle.

$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

$$\text{or } AB^2 + AC^2 = 2\left(AD^2 + \frac{BC^2}{4}\right)$$

- If two medians intersect each other at right angles, then:

$$AB^2 + AC^2 = 5BC^2$$

- If two medians of an isosceles triangle intersect each other at right angles, then:

$$\left(\frac{AB}{BC}\right)^2 = \frac{5}{2} \Rightarrow \frac{AB}{BC} = \frac{\sqrt{5}}{2}$$

Relationship between the Sides & Median of a Triangle

- The sum of any two sides of a triangle is greater than twice the median drawn to the third side.

$$AB + AC > 2AD$$

- The perimeter of a triangle (sum of its sides) is always greater than the sum of its three medians.

$$(AB + BC + CA) > (AD + BE + CF)$$

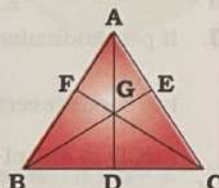
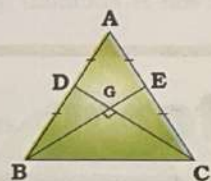
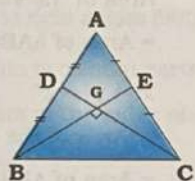
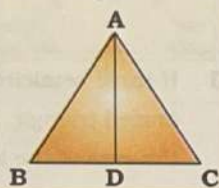
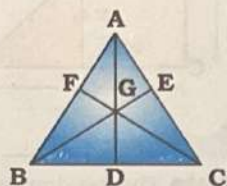
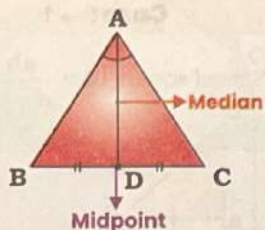
- Four times the sum of the three medians of a triangle is greater than three times the sum of its three sides.

$$3(AB + BC + CA) < 4(AD + BE + CF)$$

- Correct mathematical statement:** Three times the sum of the squares of the three sides of a triangle is equal to four times the sum of the squares of its three medians.

$$3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$$

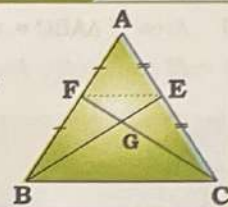
$$\text{or } \frac{AB^2 + BC^2 + CA^2}{AD^2 + BE^2 + CF^2} = \frac{4}{3} \Rightarrow 1 < \frac{AB + BC + CA}{AD + BE + CF} < \frac{4}{3}$$



Area of the Triangle Formed by the Medians

If two medians intersect each other at right angles and their lengths are given, then the area of the triangle is:

$$\text{Area of triangle} = \frac{2}{3} m_1 \times m_2$$



Properties of the Centroid in a Right-Angled Triangle

- In a right-angled triangle, the sum of the squares of the two medians and the square of the line segment joining them is equal to the square of the hypotenuse

$$AD^2 + BE^2 = DE^2 + AB^2$$

Note If D and E are the midpoints and AD and BE are medians, then $DE = \frac{AB}{2}$

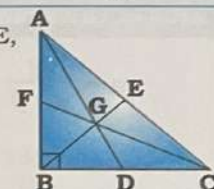
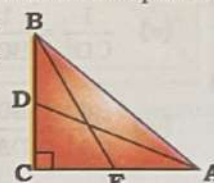
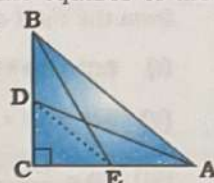
- In a right-angled triangle, four times the sum of the squares of the two medians drawn to the legs is equal to five times the square of the hypotenuse.

$$4(AD^2 + BE^2) = 5AB^2$$

$$\text{❖ } AD^2 + BE^2 = 5R^2 \quad (AB = 2R)$$

- In a right-angled triangle ABC, if AD, BE, and CF are the three medians, then:

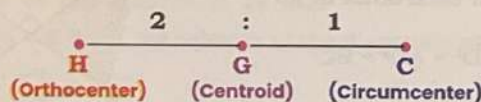
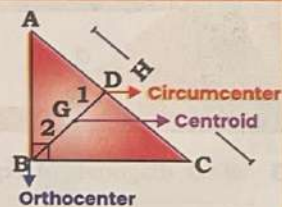
$$AG^2 + CG^2 = 5BG^2 \quad \left(BE = R = \frac{AC}{2} \right)$$



Relation between the Circumcenter, Orthocenter, and Centroid in a Right-Angled Triangle

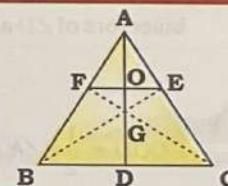
Euler's Line

In any triangle, the orthocenter, centroid, and circumcenter lie on the same straight line, called the Euler line. The centroid divides this line in the ratio 2 : 1.



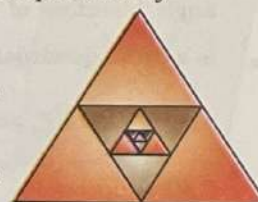
Triangle Formed by the Medians

- If O is the midpoint of the median AD, then $AO : OG : GD = 3 : 1 : 2$



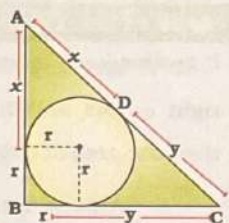
Triangle formed by joining the midpoints

- Sum of the Areas of Triangles Formed up to Infinity
- $$= \frac{4}{3} \text{ Area of the original triangle}$$
- Sum of the Perimeters of Triangles Formed up to Infinity
- $$= 2 \times \text{Perimeter of the Original Triangle}$$



Right Angle Triangle

- Area of $\triangle ABC = x \times y$



Perpendicular from the right-angle vertex to the hypotenuse

- In right-angled triangle ABC, if a perpendicular CD is drawn from the right-angled vertex to the hypotenuse,

(i) $BC^2 = BD \times BA$

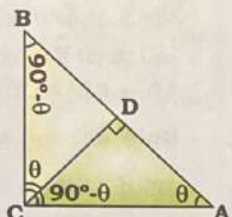
(ii) $AC^2 = AD \times AB$

(iii) $CD = \frac{BC \times AC}{AB}$

(iv) $CD^2 = BD \times AD$

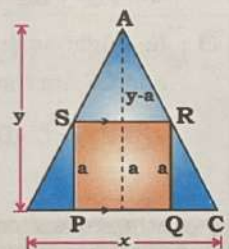
(v) $\frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{AC^2}$

(vi) $\frac{BC^2}{CA^2} = \frac{BD}{DA}$



Side of the square in any triangle

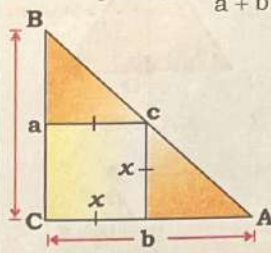
$$a = \frac{xy}{x+y}$$



- Largest possible square inside a right-angled triangle

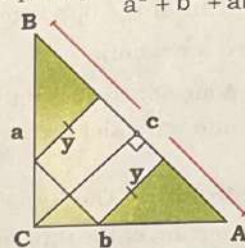
Case: $\rightarrow 1$

Side of square (x) = $\frac{ab}{a+b}$



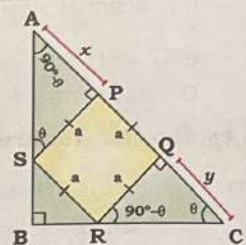
Case: $\rightarrow 2$

Side of square (y) = $\frac{abc}{a^2 + b^2 + ab}$, $x > y$



Note The maximum size of the square will be in Case 1.

Side of the square (a) = $\sqrt{xy}$



- If three semicircles are drawn on the three sides of a right-angled triangle, then the area of the shaded region is equal to the area of the triangle.

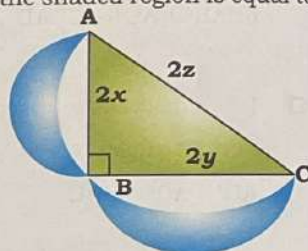
Area of the shaded region

= Area of $\triangle ABC$

$$x^2 + y^2 = z^2$$

$\therefore$ Area of the shaded region

= Area of $\triangle ABC$



Quadrilateral

- If the diagonals of a quadrilateral intersect each other at 90° ,

$$AB^2 + CD^2 = AD^2 + BC^2$$

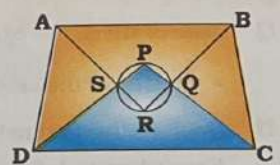
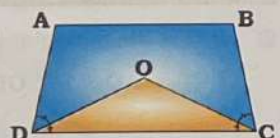
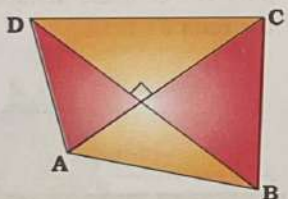
- If DO and CO are the angle bisectors of $\angle D$ and $\angle C$ respectively, then

$$\angle COD = \frac{1}{2} (\angle A + \angle B)$$

- The quadrilateral formed by the angle bisectors of a quadrilateral is a cyclic quadrilateral.

$$\angle P + \angle R = 180^\circ, \text{ and } \angle S + \angle Q$$

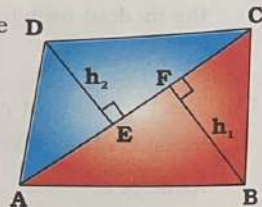
$$= 180^\circ$$



Area and Perimeter of a Quadrilateral

- If perpendiculars are drawn from the two opposite vertices of a quadrilateral ABCD to one of its diagonals,

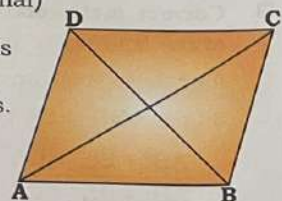
then find the area of the quadrilateral.



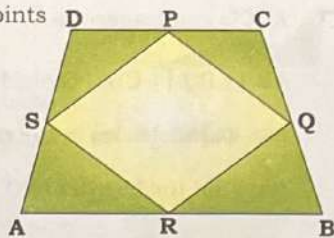
$$\text{Area} = \frac{1}{2} \times AC \times (h_1 + h_2) = \frac{1}{2} \times \text{Diagonal} \times (\text{Sum of the perpendiculars drawn to the diagonal})$$

- The perimeter of a quadrilateral is greater than the sum of its diagonals.

$$(AB + BC + CD + DA) > (AC + BD)$$



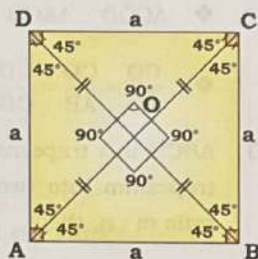
- If P, Q, R, and S are the midpoints of the sides of a quadrilateral ABCD and are joined to form a new quadrilateral, then the new quadrilateral will be a parallelogram.



$$\text{Area of PQRS} = \frac{\text{Area of } \square ABCD}{2}$$

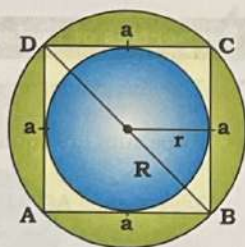
Square

A square is a quadrilateral whose all four sides are equal and each angle is 90° (a right angle).



Properties of a Square

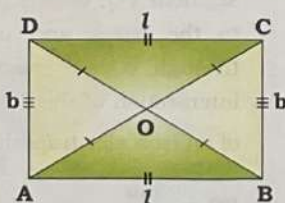
- All sides are equal, and opposite sides are parallel.
- Each angle is a right angle (90°).
- The diagonals are equal & bisect each other at right angles (90°).
- The diagonals of a square divide its area into two equal parts.
- The figure formed by joining the midpoints of the sides of a square is also a square.
- Every square is a rhombus, but every rhombus is not a square.
- Perimeter = $4a$
- Length of the diagonal ($AC = BD = 'd'$)
 $= \sqrt{2}a$
- Area = Side $\times$ Side = $a^2 = \frac{d^2}{2}$
- Radius of the incircle (r) = $\frac{a}{2}$
- Radius of the circumcircle $R = \frac{d}{2} = \frac{a}{\sqrt{2}}$
- Ratio of the radii of the incircle & circumcircle ($r : R$) = $1 : \sqrt{2}$
- Ratio of the areas of the incircle & circumcircle ($r^2 : R^2$) = $1 : 2$



Rectangle

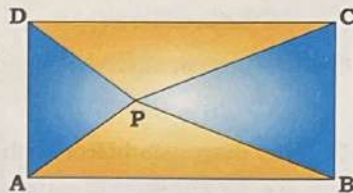
A rectangle is a quadrilateral whose all four angles are right angles (90°) and whose opposite sides are equal and parallel.

Where, l = length, b = breadth



Properties of a Rectangle

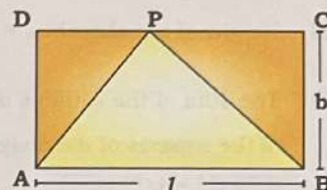
- Opposite sides are parallel and equal.
- Both diagonals are equal and bisect each other, but they do not intersect at right angles.
- The diagonals do not bisect the angles.
- Perimeter = $2(l + b)$
- Area = $l \times b$
- Diagonal = ($AC = BD$) = $\sqrt{l^2 + b^2}$
- If there is a point inside a rectangle or square,
 $PA^2 + PC^2 = PB^2 + PD^2$



- In a rectangle ABCD, if P is any point on side DC, then (P may not be the midpoint of DC.)

Area of $\triangle APB$

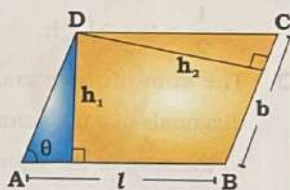
$$= \frac{\text{Area of } \square ABCD}{2}$$



Note: This result is true for a rhombus, parallelogram, & square.

Parallelogram

- Opposite sides are equal and parallel.
- Opposite angles are equal.
- The sum of any two adjacent angles is 180° .
- The diagonals bisect each other.
- The diagonals are not equal in length.
- The diagonals do not bisect each other at right angles.
- Perimeter = $2(l + b)$
- Area = Base $\times$ Height
 $\diamond = l \times h_1 = b \times h_2$
 $\diamond = AD \times AB \sin \theta$
- Sum of the squares of all four sides = Sum of the squares of the diagonals $\Rightarrow d_1^2 + d_2^2 = 2(l^2 + b^2)$
- Ratio of angles = $\angle A : \angle B : \angle C : \angle D = a : b : a : b$
- The diagonals divide the quadrilateral into four equal-area triangles.



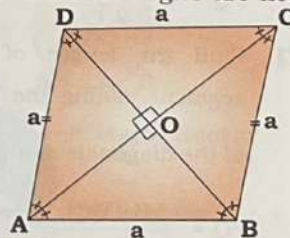
Rhombus

A quadrilateral whose all four sides are equal and whose diagonals bisect each other at right angles, but whose all angles are not right angles, is called a rhombus.

Diagonal (AC) = d_1 ,

Diagonal (BD) = d_2

Side of the rhombus = a



Properties of a Rhombus

- Opposite sides are parallel and equal.
- Opposite angles are equal.
 $\angle A = \angle C, \angle B = \angle D$
- The sum of adjacent angles of a rhombus is 180° .
 $\angle A + \angle B = 180^\circ$
- The diagonals bisect each other at right angles, but they are not necessarily equal.
- A rhombus may or may not be a square, but every square is a rhombus.
- Perimeter = $4a$
- Area = $\frac{1}{2} \times d_1 \times d_2 = a^2 \sin \theta$
- Height of the rhombus = $\frac{d_1 \times d_2}{\sqrt{d_1^2 + d_2^2}}$
- The sum of the squares of all four sides is equal to the sum of the squares of the diagonals.
 $d_1^2 + d_2^2 = 4a^2$
- The figure formed by joining the midpoints of the sides of a rhombus is a rectangle.

Trapezium

- Area of a Trapezium

$$= \frac{1}{2} \times \text{Sum of parallel sides}$$

× Distance between the parallel sides

Distance between them

$$= \frac{1}{2} \times (a + b) \times h$$

- The sum of the squares of the diagonals of a trapezium is equal to the sum of the squares of the non-parallel sides plus twice the product of the parallel sides.

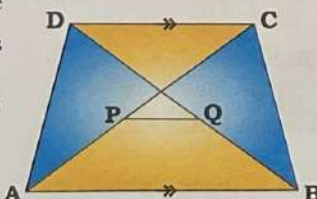
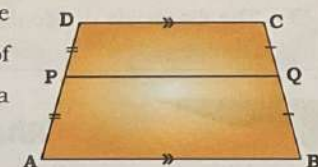
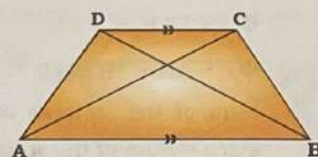
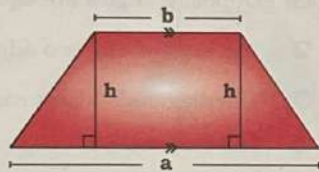
$$AC^2 + BD^2 = AD^2 + BC^2 + 2 AB \times CD$$

- Find the length of the line segment joining the midpoints of the non-parallel sides of a trapezium.

$$PQ = \frac{(AB + DC)}{2}$$

- Find the length of the line segment joining the midpoints of the diagonals of a trapezium.

$$PQ = \frac{(AB - DC)}{2}$$



- ABCD is a trapezium in which $AB \parallel PQ \parallel CD$, PQ divides the non-parallel sides in the ratio $m : n$. Find the length of PQ.

$$PQ = \frac{ma + nb}{m + n}$$

- ABCD is a trapezium in which the diagonals AC and BD intersect each other at O.

$$\diamond \triangle COD \sim \triangle AOB$$

$$\diamond \frac{CO}{AO} = \frac{CD}{AB} = \frac{OD}{OB}$$

- ABCD is a trapezium in which PQ is a line that divides the trapezium into two quadrilaterals whose areas are in the ratio $m : n$, then

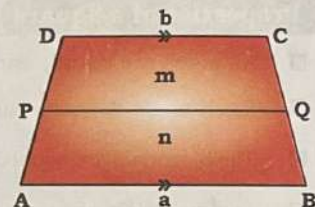
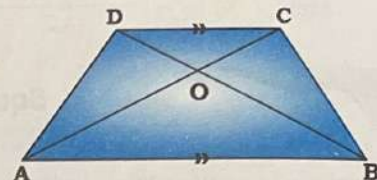
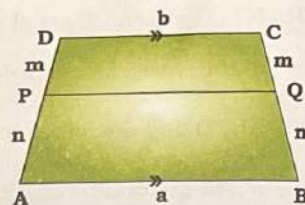
$$\diamond PQ = \sqrt{\frac{ma^2 + nb^2}{m + n}}$$

- If $m = n$, then PQ

$$= \sqrt{\frac{a^2 + b^2}{2}}$$

- $AB \parallel PQ \parallel CD$ then

$$\frac{\text{Area of } \square PQCD}{\text{Area of } \square PQBA} = \frac{m}{n}$$



Properties of an Isosceles Trapezium

- The diagonals of every isosceles trapezium are equal.

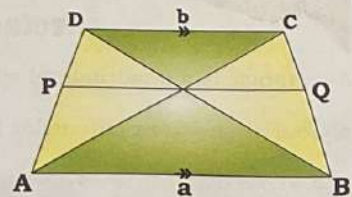
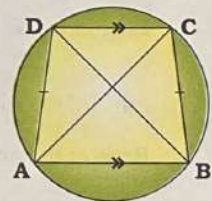
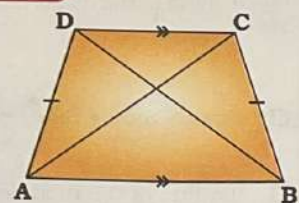
$$\text{Diagonal} \Rightarrow AC = BD$$

- Every isosceles trapezium is a cyclic quadrilateral.

- If a trapezium is inscribed in a circle, then it is an isosceles trapezium.

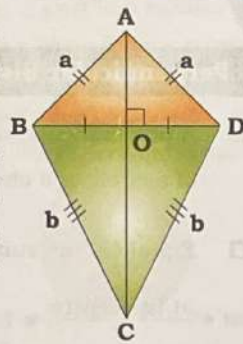
- Find the length of the line segment PQ, drawn parallel to the bases and passing through the point of intersection of the diagonals of an isosceles trapezium.

$$PQ = \frac{2ab}{a + b}$$



Kite

- A quadrilateral in which two distinct pairs of adjacent sides are equal is called a kite.
- The diagonals of a kite intersect each other at right angles, and one diagonal bisects the other. The opposite angles formed between the unequal sides are equal.



Properties of a Kite

- $AB = AD = a$
 $BC = CD = b$
- $\angle B = \angle D$, ($\angle A \neq \angle C$)
- Perimeter = $2(a + b)$
- Area = $\frac{1}{2} \times d_1 \times d_2$
- The diagonals intersect each other at right angles.
 $AC \perp BD$
- The longer diagonal bisects the shorter diagonal.
 $BO = OD$
- The diagonal bisects the angles into two equal parts.
 $\angle BAC = \angle DAC$ and $\angle BCA = \angle DCA$

Figure formed by the midpoints of a quadrilateral

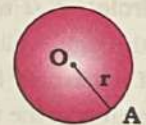
Name	Given figure	Figure formed by consecutive midpoints
Quadrilateral		Parallelogram
Parallelogram		Parallelogram
Rectangle		Rhombus
Square		Square
Rhombus		Rectangle
Trapezium		Parallelogram
Kite		Rectangle

Circle

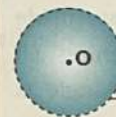
Important Definition

Definition	Figure
Circle: The locus of all points that are at the same distance from a fixed point is called a circle. The fixed point is called the centre of the circle.	
Arc: A continuous part of a circle is called an arc.	
Diameter: A chord passing through the centre of a circle is called the diameter. A diameter is a line segment passing through the centre, with both endpoints on the circumference of the circle. It is the longest chord of a circle.	

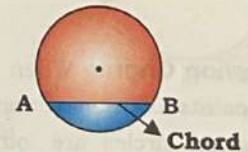
Radius: The distance between the centre of a circle and any point on its circumference is called the radius of the circle.
Radius of the circle = r



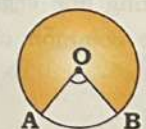
Circumference: It is the length of the boundary of a circle and is equal to $2\pi r$



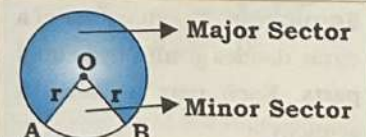
Chord: A line segment joining any two points on the circumference of a circle is called a chord. A longer chord lies closer to the centre of the circle.



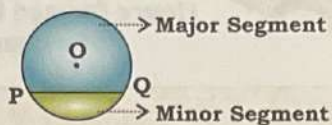
Central Angle: An angle formed at the centre of a circle is called a central angle.



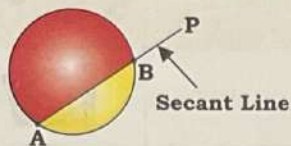
Sector: The region enclosed by two radii and the corresponding arc is called a sector.



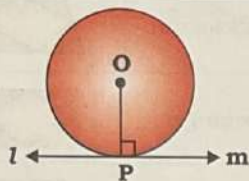
Segment: The region of a circle bounded by a chord and its corresponding arc is called a segment.



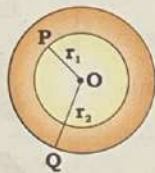
Secant: A straight line that intersects a circle at two points is called a secant of the circle.



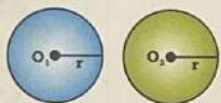
Tangent: A line that touches a circle at only one point is called a tangent to the circle. $\angle OPM = 90^\circ$. If $l \parallel m$ is tangent to the circle at P, then $OP \perp l \parallel m$.



Concentric Circles: When two circles in the same plane have the same centre, they are called concentric circles. Here, the two circles with common centre O and radii r and R are concentric circles.



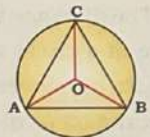
Congruent Circles: When two different circles have equal radii, they are called congruent circles.



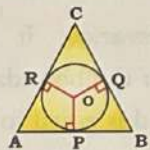
Cyclic Quadrilateral: A quadrilateral whose all four vertices lie on a circle is called a cyclic quadrilateral. ABCD is a cyclic quadrilateral.



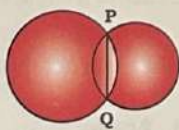
Circumcircle: A circle that passes through all three vertices of a triangle is called the circumcircle of the triangle.



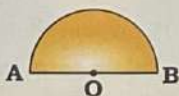
Incicle: A circle drawn inside a triangle and touching all three sides of the triangle is called an incircle.



Common Chord: When the two points of intersection of two given circles are joined, the resulting line segment is called the common chord of both circles.



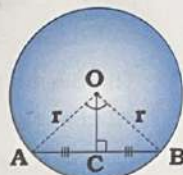
Semicircle: The diameter of a circle divides it into two equal parts. Each part is called a semicircle.



Important Properties of Chords and Arcs

Perpendicular Bisector Theorem

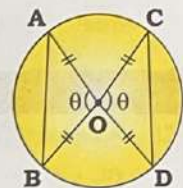
- A perpendicular drawn from the centre of a circle to a chord bisects the chord.



- Equal chords subtend equal angles at the centre.

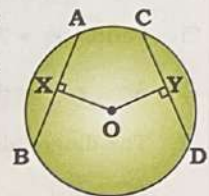
If $AB = CD$ then $\angle AOB = \angle COD$

Or if $\angle AOB = \angle COD$ then $AB = CD$



- If two chords are equidistant from the centre of a circle, then the two chords are equal.

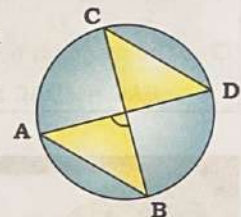
If $AB = CD$, $OX \perp AB$ then $OY \perp CD$, then $OX = OY$



- If two chords of a circle are equal, then their corresponding arcs are also equal.

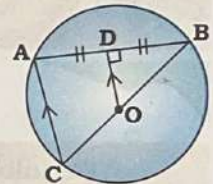
Arc $(AB) = \text{Arc } (CD)$

$\therefore AB = CD$



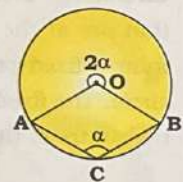
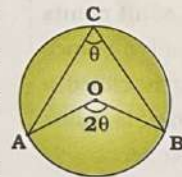
- If, in a circle, OD is perpendicular to chord AB, O is the centre of the circle, and BC is a diameter.

then $\frac{CA}{OD} = \frac{2}{1}$

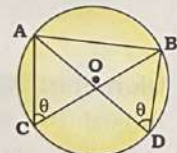
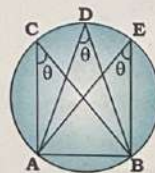


- The angle subtended by an arc at the centre, the angle subtended by the same arc.

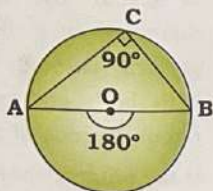
The angle subtended at the centre is twice the angle subtended at the circumference by the same arc.



- Angles subtended by the same arc in the same segment of a circle are equal.

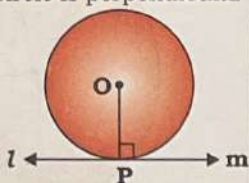


- If the two endpoints of a diameter are joined to any point on the semicircle, the angle formed at that point is always 90° .



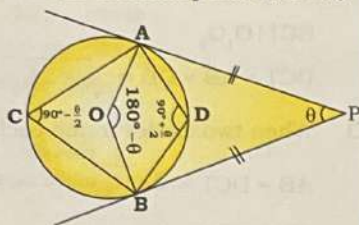
Tangent Line

- An infinite number of tangents can be drawn to a circle.
- At most two tangents can be drawn to a circle from a point outside the circle.
- Only one tangent can be drawn at a point on a circle.
- The tangent drawn at any point of a circle is perpendicular to the radius passing through the point of contact.
- A line drawn perpendicular to a radius at its endpoint on the circle is a tangent to the circle.



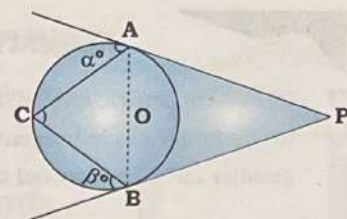
When two tangents are drawn from an external point P, then,

- $\angle ADB = 90^\circ + \frac{\theta}{2}$
- $\angle ACB = 90^\circ - \frac{\theta}{2}$
- $\angle APB = \theta$
- $\angle AOB = 180^\circ - \theta$
- The lengths of the tangents drawn from an external point to a circle are equal.

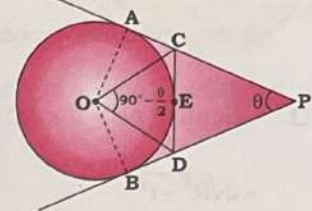


- If α and β are given.
Then find the value of $\angle ACB$

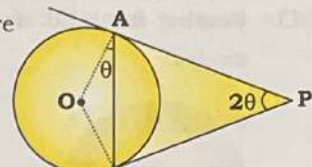
$$\angle ACB = 180^\circ - (\alpha + \beta)$$



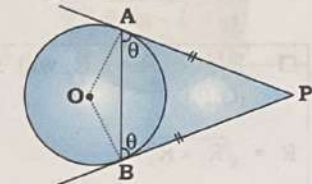
- $\angle COD = 90^\circ - \frac{\theta}{2}$
- $\angle COD = \frac{\angle AOB}{2}$



- If two tangents AP and PB are drawn from an external point P to a circle with centre O, then,
 $\angle APB = 2\angle OAB$



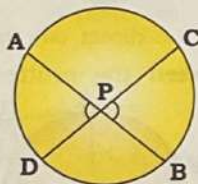
- If the tangents drawn at the endpoints of a chord from an external point make equal angles with the chord, then, $PA = PB$
 $\angle PAB = \angle PBA$



Relationship between Tangent, Secant and Chord

Two chords intersecting inside a circle.

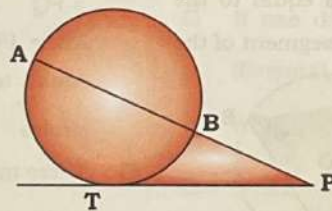
- Two chords AB and CD intersect internally at point P.



$$PA \times PB = PC \times PD$$

When a chord and a tangent meet.

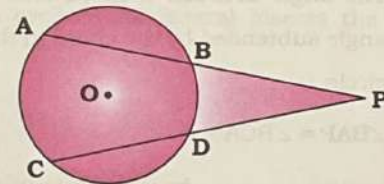
- If a secant and a tangent are drawn from an external point, then,



$$PT^2 = PB \times PA$$

Two secants intersecting outside a circle.

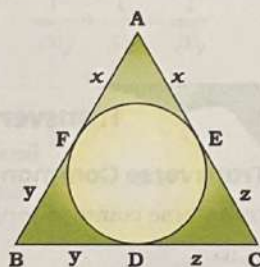
- Two chords AB and CD intersect externally at point P.



$$PB \times PA = PD \times PC$$

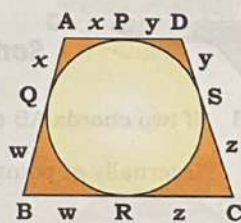
- When a circle lies inside a triangle.

$$AB + BC + CA = 2(x + y + z)$$



- If a circle is inscribed in a quadrilateral, or a quadrilateral is circumscribed about a circle.

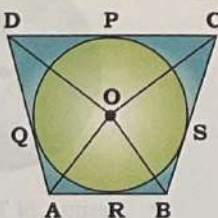
$$AB + CD = BC + AD$$



- If a circle touches the sides of a quadrilateral ABCD at P, Q, R and S respectively, then the angles subtended at the centre by opposite sides are supplementary.

$$\angle AOB + \angle COD = 180^\circ$$

$$\angle AOD + \angle BOC = 180^\circ$$



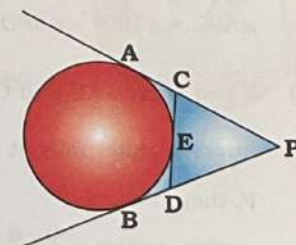
- Perimeter of $\triangle PCD = 2PA = 2PB$

$$\Rightarrow PC + CE + ED + PD$$

$$\Rightarrow PC + CA + BD + PD$$

$$\Rightarrow PA + PB$$

$$\Rightarrow 2PA \text{ or } 2PB (\because PA = PB)$$



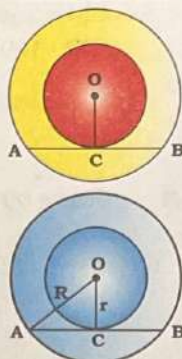
Tangent in Concentric

- In two concentric circles, a chord of the larger circle tangent to the smaller circle is bisected at the point of contact, i.e., $AP = BP$.

If the circles are concentric, then:

- Length of $AB = 2 \times AC$

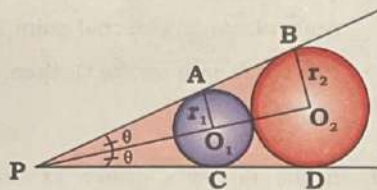
$$= 2\sqrt{R^2 - r^2}$$



- Finding the ratio of the radii of two externally touching circles.

$$O_1 O_2 = r_1 + r_2$$

$$\frac{r_1}{r_2} = \frac{1 - \sin\theta}{1 + \sin\theta}$$

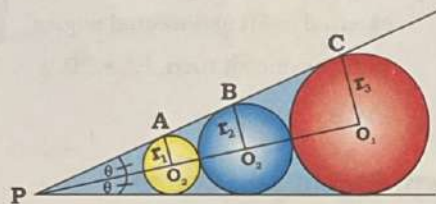


- R_1, R_2 , and R_3 will always be in geometric progression (G.P.).

$$R_2 = \sqrt{R_1 \times R_3}$$

$$\frac{1 - \sin\theta}{1 + \sin\theta} = \frac{R_1}{R_2} = \frac{R_2}{R_3}$$

$$\Rightarrow R_2 = \sqrt{R_1 \times R_3}$$



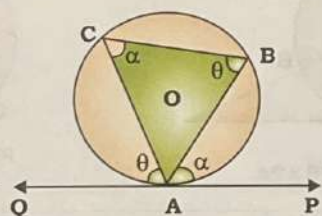
Alternate Segment Theorem

- The angle between a chord and a tangent is equal to the angle subtended by the chord in the alternate segment of the circle.

$$\angle BAP = \angle BCA = \alpha$$

$$\angle CAQ = \angle CBA = \theta$$

- It is also known as the Chord-Tangent Theorem.

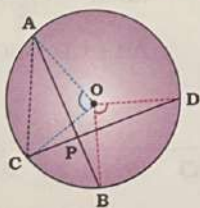


Some Important Results

- If two chords AB and CD intersect internally at point P, then,

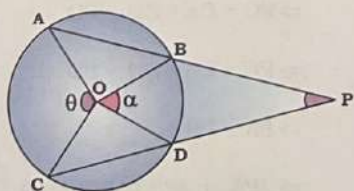
$$\angle APD = \angle BPC = \frac{\angle AOD + \angle BOC}{2}$$

$$\angle APC = \angle 180^\circ - \angle APD = \angle BPD$$



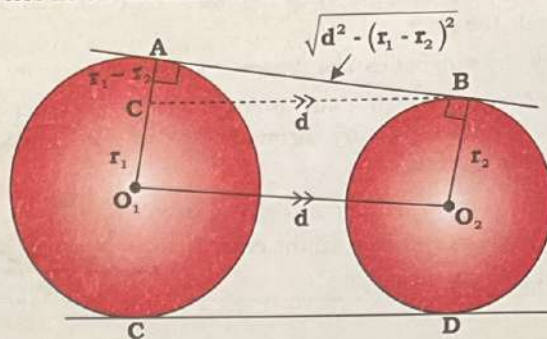
- If two chords AB and CD intersect externally at point P, then,

$$\angle APC = \angle BPD = \frac{\theta - \alpha}{2}$$



Length of Common Tangents

- A common tangent is called a direct common tangent if both circles lie on the same side of it.



Length of DCT

$$BC \parallel O_1 O_2$$

$$DCT = AB = CD = \sqrt{d^2 - (r_1 - r_2)^2}$$

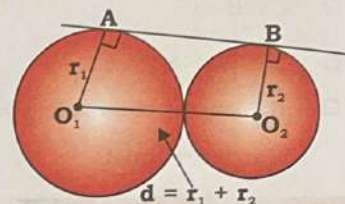
- When two circles touch each other externally, then:

$$AB = DCT = 2\sqrt{r_1 r_2}$$

$$AB = DCT$$

$$= \sqrt{(r_1 + r_2)^2 - (r_1 - r_2)^2}$$

$$= \sqrt{4r_1 r_2} = 2\sqrt{r_1 r_2}$$



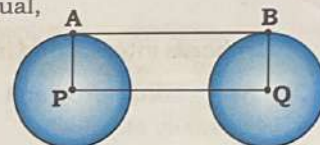
- If the radii of the circles are equal,

$$r_1 = r_2 = r$$

Direct Common Tangent, AB

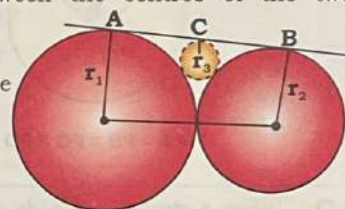
$$= PQ$$

$AB = PQ$ i.e., the length of the direct common tangent is equal to the distance between the centres of the two circles.



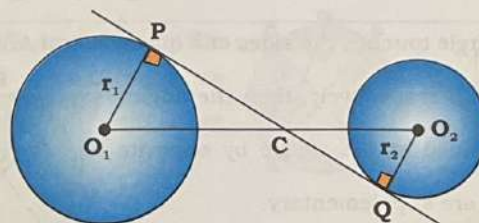
- Three mutually touching circle with a common tangent

$$\frac{1}{\sqrt{r_3}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}}$$



Transverse Common Tangent

Transverse Common Tangent: A common tangent is called a transverse common tangent if the two circles lie on opposite sides of it.



❖ Length of Transverse Common Tangent (TCT)

$$(PQ) = \sqrt{d^2 - (r_1 + r_2)^2} \text{ (Here, } d = O_1 O_2 \text{)}$$