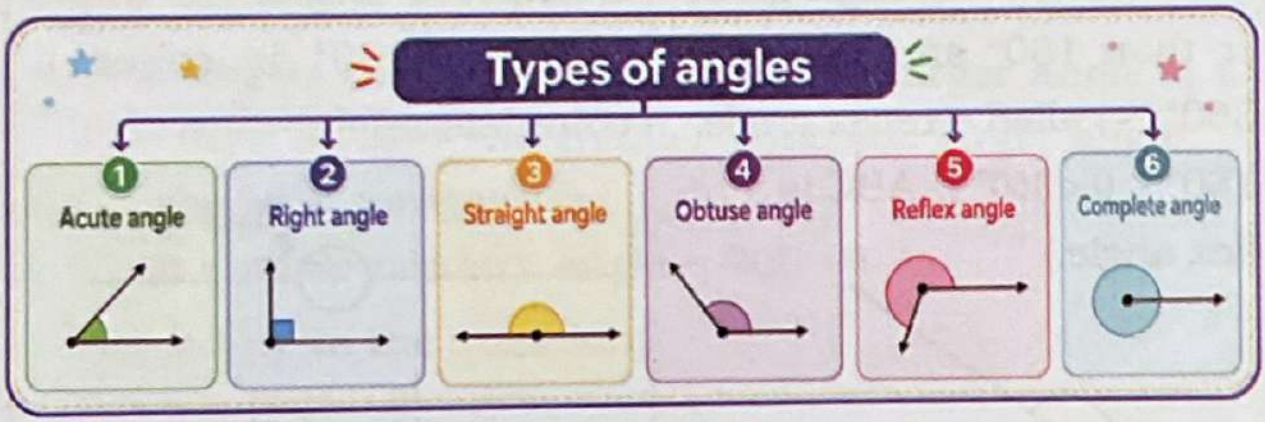
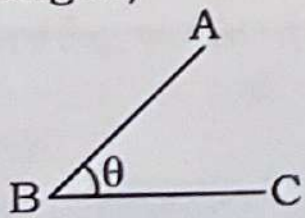
Lines & Angles

Angle: The inclination between two lines is called an angle. A figure formed by two rays or lines that share a common end point is called an angle. The two rays are called the arms of the angle and the common end point is called the vertex.



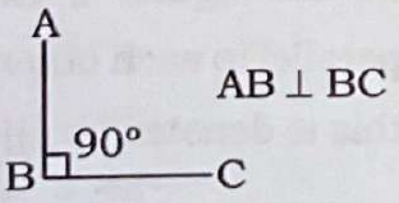
Acute angle: An angle greater than 0° and less than 90° is called an acute angle.

$0° < \theta < 90°$ ($\angle$ ABC is an acute angle.)



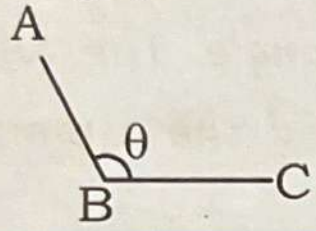
Right angle: An angle equal to 90° is called a right angle.

$\theta = 90°$ ($\angle$ABC is a right angle.)



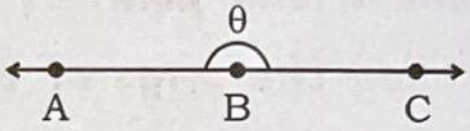
Obtuse angle: An angle greater than 90° and less than 180° is called an obtuse angle.

$90° < \theta < 180°$ ($\angle ABC$ is an obtuse angle).



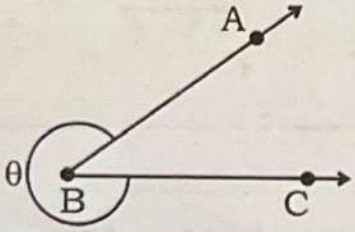
Straight angle: An angle equal to 180° is called a straight angle.

$\theta = 180°$ ($\angle ABC$ is a straight angle).

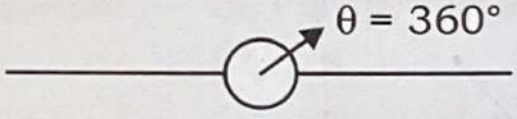


Reflex angle: An angle greater than 180° and less than 360° is called a reflex angle.

$180° < \theta < 360°$ ($\angle ABC$ is a reflex angle.)

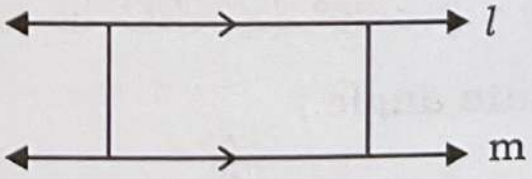


Complete angle: An angle equal to 360° is called a complete angle.

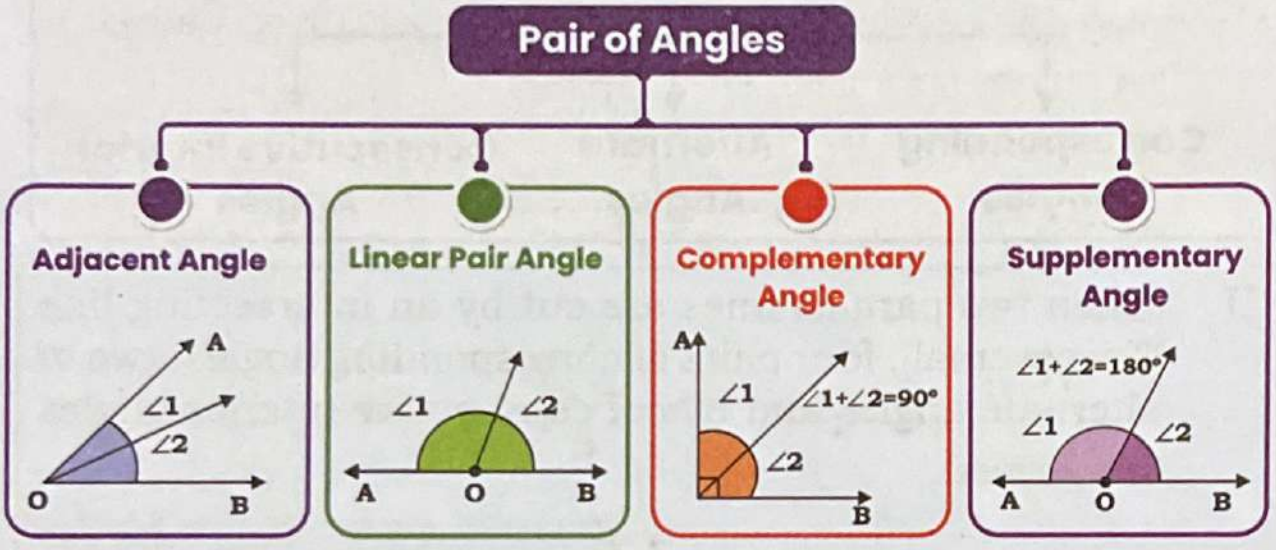


❐ **Parallel lines:** Two or more lines that never intersect each other are called parallel lines.

In this figure, l, m are parallel to each other and this is denoted by $l \parallel m$.



Pair of angles



Ex. Two angles are complementary. The larger angle is 6° less than 5 times the smaller. Find the larger angle.

Sol. Let the larger angle = θ

Then complementary angle = $90° - \theta$

According to the question,

$\theta = 5 \times (90° - \theta) - 6°$

$\theta = 450° - 5\theta - 6°$

$6\theta = 444°$ $\quad\therefore \theta = 74°$

Ex. The supplement of an angle is 10° more than four times the angle. Find the complement of that angle.

Sol. Let the original angle = θ

Supplementary angle = $180° - \theta$

According to the question,

$\theta \times 4 + 10° = 180° - \theta$

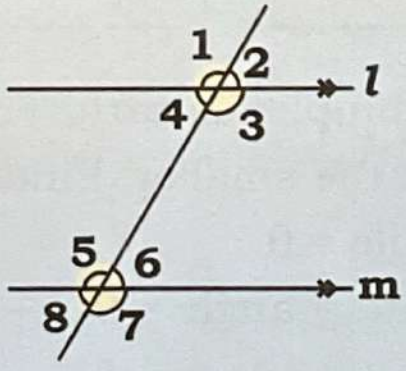
$\theta = 34°$

Complementary angle = $90° - 34° = 56°$

Angles Formed by a Transversal

- Corresponding Angles
- Alternate Angles
- Consecutive Interior Angles

❒ When two parallel lines are cut by an intersecting line (Transversal), four pairs of corresponding angles, two of alternate angles and two of consecutive interior angles are formed.



- ❖ **Corresponding angles:** $\angle 1 = \angle 5, \angle 2 = \angle 6, \angle 3 = \angle 7, \angle 4 = \angle 8$
- ❖ **Alternate interior angles:** $\angle 4 = \angle 6, \angle 3 = \angle 5$
- ❖ **Alternate exterior angles:** $\angle 1 = \angle 7, \angle 2 = \angle 8$
- ❖ **Consecutive interior angles:** $\angle 4 + \angle 5 = 180, \angle 3 + \angle 6 = 180°$

Ex. If a transversal cuts two parallel lines and the difference of the two interior angles on the same side of the transversal is 40°, then find the smallest of the interior angles.

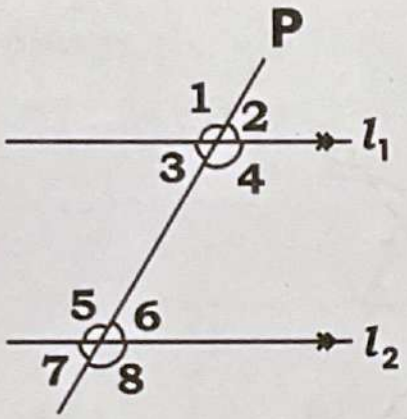
Sol. $\theta_1 + \theta_2 = 180°$ .....(i)

$\theta_1 - \theta_2 = 40°$ .....(ii)

From equations (i) and (ii)

$\theta_1 = 110°, \theta_2 = 70°$

Ex. In the given figure, $l_1 \parallel l_2$ and p is their transversal, where $\angle 4 = 5x + 35°$ and $\angle 7 = 45°$. Find the value of x.



Sol. $\angle 4 = 5x + 35°$

$\angle 7 = 45°$

$\angle 7 = \angle 3 = 45°$

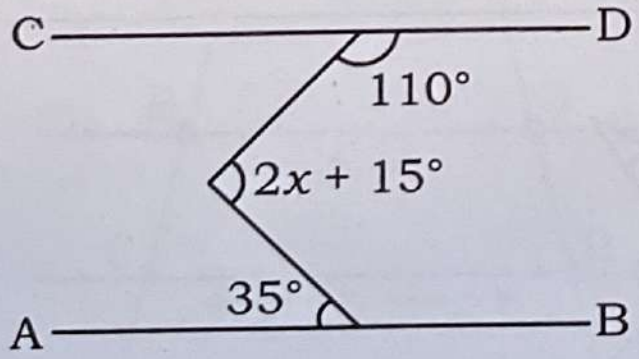
$\angle 4 + \angle 3 = 180°$

$\Rightarrow 5x + 35° + 45° = 180°$

$\Rightarrow 5x = 100°$

$\Rightarrow x = 20°$

Ex. In the figure given below, AB || CD. Find the value of x.

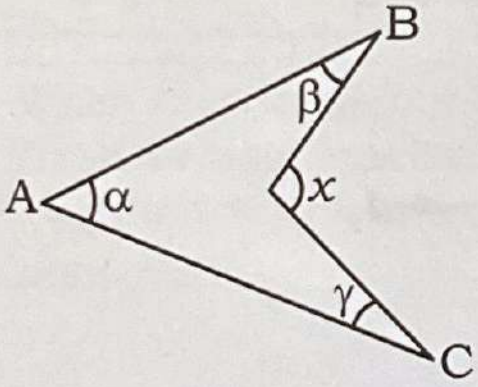


Sol. $2x + 15° = 35° + 70°$

$\Rightarrow x = 45°$

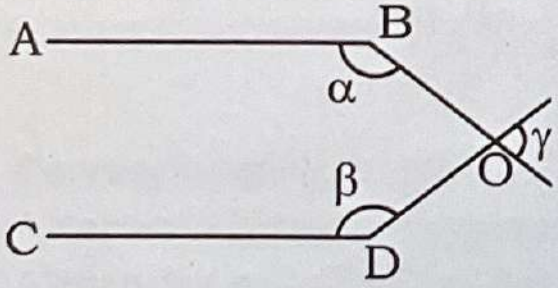
❒ Based on Scissor Figure





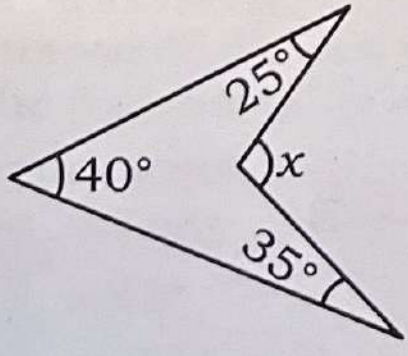
$x = \alpha + \beta + \gamma$

❒ If AB || CD then find the value of $\alpha + \beta + \gamma$



$\alpha + \beta + \gamma = 360°$

Ex. x = ?



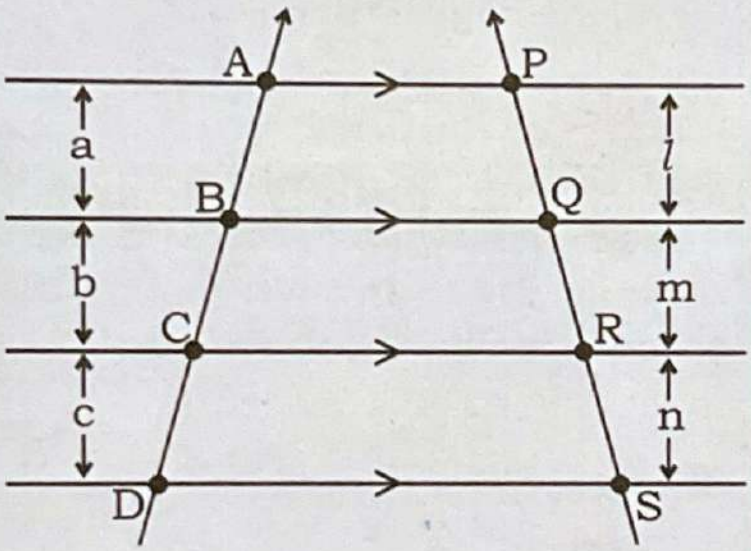
Sol. $x = 40° + 25° + 35° = 100°$

Some Important results

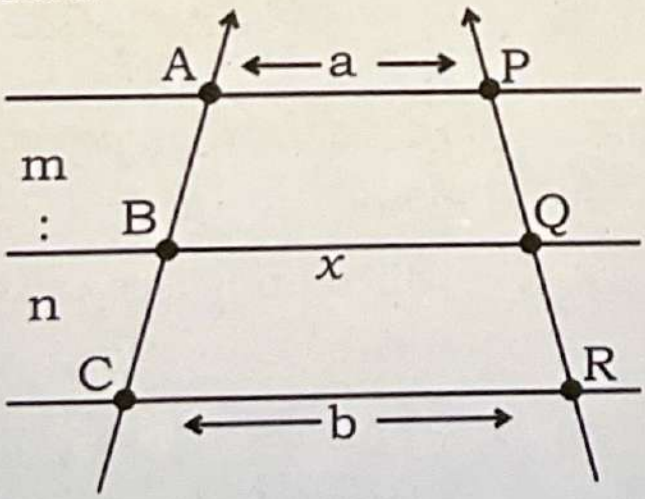
❒ When AP || BQ || CR || DS and these lines are cut by two transversals.

Then, $a : b : c = l : m : n$

or $\dfrac{a}{a+b+c} = \dfrac{l}{l+m+n}$



When AP || BQ || CR and these lines are cut by two transversals.



$$x = \frac{mb + na}{m+n}$$

Triangle

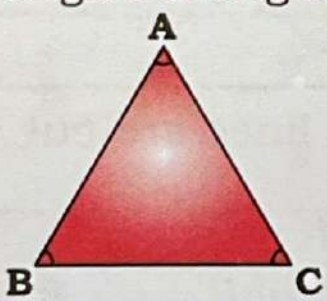
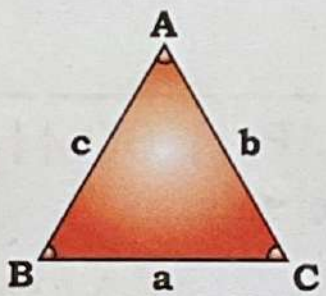
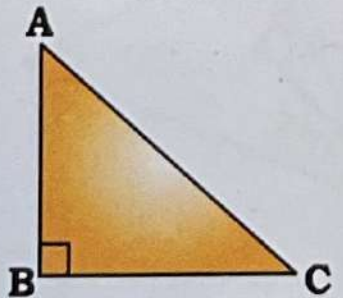
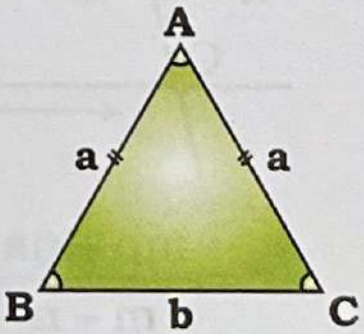
Types of Triangles

Based on Angles

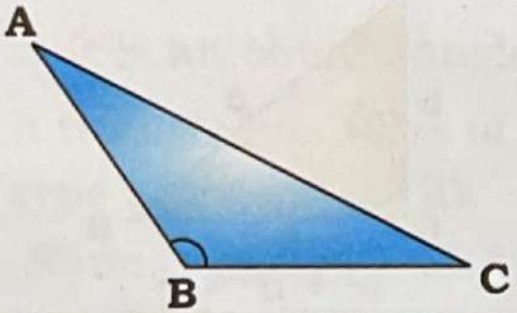
- Acute-angled Triangle
- Right-angled Triangle
- Obtuse-angled Triangle

Based on Sides

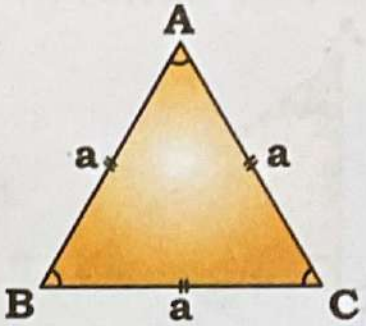
- Scalene Triangle
- Isosceles Triangle
- Equilateral Triangle

Triangles on the Basis of Angles	Triangles on the Basis of Sides
Acute-angled Triangle: A triangle in which each angle is less than 90° is called an acute-angled triangle. 	**Scalene Triangle:** A triangle whose all three sides are different is called a scalene triangle. 
Right-angled Triangle: A triangle in which one angle is 90° is called a right-angled triangle. 	**Isosceles Triangle:** A triangle whose two sides are equal is called an isosceles triangle. 

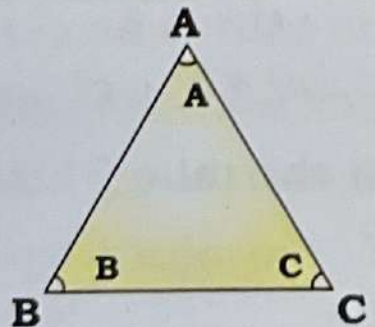
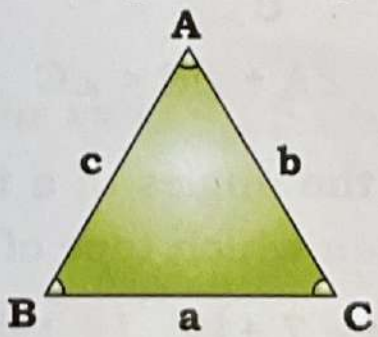
Obtuse-angled Triangle: A triangle in which one angle is more than 90° is called an obtuse-angled triangle.



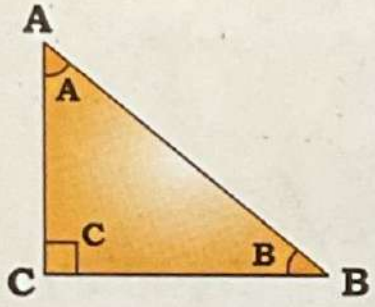
Equilateral Triangle: A triangle with all sides equal is called an equilateral triangle.



Identifying a Triangle by the Ratio of Angles and Sides

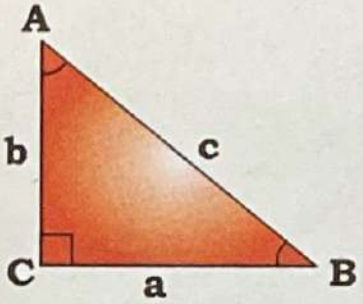
On the Basis of Ratio of Angles	On the Basis of Ratio of Sides
If the sum of the ratio terms of two angles of a triangle is greater than the third angle, then the triangle is called an acute-angled triangle.	If the sum of the squares of two sides of a triangle is greater than the square of the largest side, then the triangle is called an acute-angled triangle.
 $\angle A + \angle B > \angle C$	 $a^2 + b^2 > c^2$

If the sum of two angles is equal to the third angle, then the triangle is a right-angled triangle.



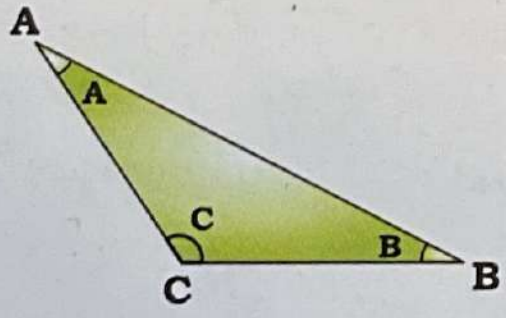
$\angle A + \angle B = \angle C$

If the sum of the squares of two sides of a triangle is equal to the square of the largest side, then the triangle is called a right-angled triangle.



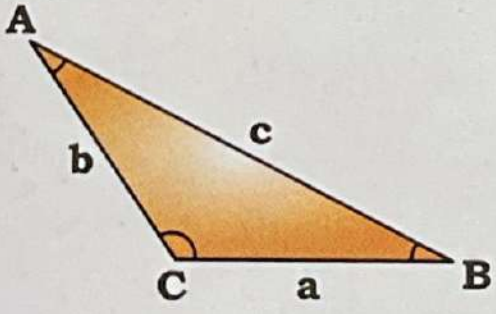
$a^2 + b^2 = c^2$

If the sum of the ratio terms of two angles of a triangle is less than the third angle, then the triangle is called an obtuse-angled triangle.



$\angle A + \angle B < \angle C$

If the sum of the squares of two sides of a triangle is less than the square of the largest side, then the triangle is called an obtuse-angled triangle.



$a^2 + b^2 < c^2$

Ex. If the angles of a triangle are in the ratio 7 : 12 : 19, then which type of triangle is it?

Sol. $19 = 7 + 12$

$\therefore$ It is a right-angled triangle.

Ex. If the angles of a triangle are in the ratio 10 : 13 : 18, then which type of triangle is it?

Sol. $18 < 10 + 13$

$\therefore$ It is an acute-angled triangle.

Ex. If the angles of a triangle are in the ratio 13 : 20 : 41, then which type of triangle is it?

Sol. $41 > 13 + 20$

$\therefore$ It is an obtuse-angled triangle.

Ex. A triangle has sides of length 13.3, 19, and 24.7; which type of triangle is it?

Sol. Ratio of sides $\rightarrow$ 13.3 : 19 : 24.7

$\rightarrow 7 : 10 : 13$

$13^2 > 7^2 + 10^2$

$169 > 149$

$\therefore$ It is an obtuse-angled triangle.

Ex. If the three altitudes of a triangle are in the ratio 3 : 4 : 5, then what type of triangle will it be?

Sol. $a : b : c = \frac{1}{h_1} : \frac{1}{h_2} : \frac{1}{h_3} \Rightarrow \frac{1}{3} : \frac{1}{4} : \frac{1}{5} = 20 : 15 : 12$

$20^2 > 15^2 + 12^2$

$400 > 369$

$\therefore$ It is an obtuse-angled triangle.

Ex. If the sides of a triangle are in the ratio 4 : 5 : 6, then what type of triangle will it be?

Sol. Largest side $\rightarrow 6$

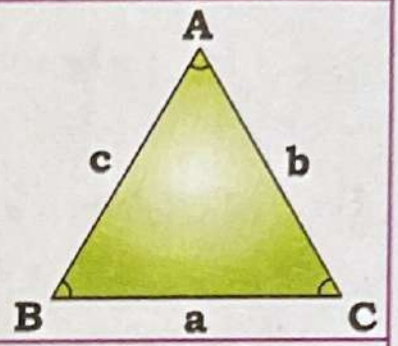
$6^2 < 4^2 + 5^2$ $\qquad 36 < 41$

$\therefore$ It is an acute-angled triangle.

Triangle Inequality

- A triangle can be formed only when the sum of two sides is greater than the third side

  $a + b > c,\ b + c > a,\ c + a > b$



- If the difference of two sides is less than the third side, then a triangle will be formed.

  $|a - b| < c,\ |b - c| < a,\ |c - a| < b$

  $\therefore\ |c - a| < b < (c + a)$

Ex. A triangle has sides 19 cm, 25 cm and y cm. Which option most accurately gives the possible values of y?

(a) $6 \le y \le 44$ (b) $6 < y \le 44$

(c) $6 \le y < 44$ (d) $6 < y < 44$

Sol. (d) $|25-19| < y < (25 + 19)$

$= 6 < y < 44$

Ex. 2 cm, 5 cm, 7 cm and 8 cm long line segments are given. How many triangles can any 3 of them form?

Sol. (2, 7, 8), (5, 7, 8)

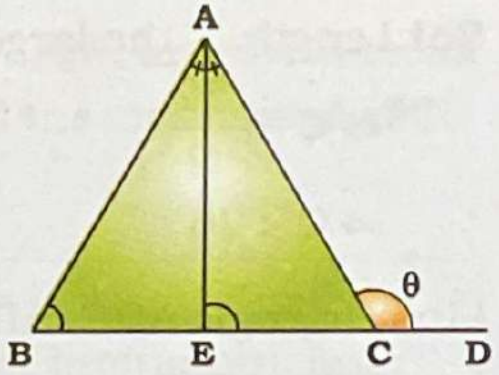
Therefore, two triangles are possible.

Ex. If the lengths of two sides of an isosceles triangle are 6 cm and 12 cm, then find the length of the third side.

(a) 8 cm (b) 6 cm

(c) 12 cm (d) 14 cm

Sol. $|c - a| < b < (c + a)$



First case: Let the equal side be 6, then $6 + 6 = 12$ (so it will not form a triangle).

Second case: Let the equal side be 12, then $(12 + 12) > 6$, $(12 + 6) > 12$ (so it will form a triangle).

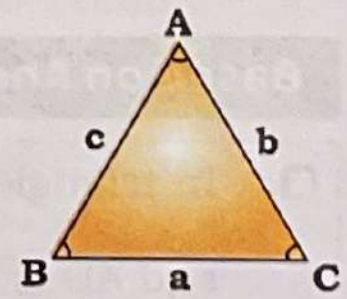
Therefore, length of the third side = 12 cm

Ex. If two sides of a triangle are 876 cm and 678 cm and the third side is an integer, then find the number of possible triangles.

Sol. Number of triangles = 2 × value of smaller side − 1

Number of triangles = $678 \times 2 - 1 = 1355$

❐ The length of the largest side of a triangle is less than half of its perimeter.



$c < (a + b)$

$(c + c) < (a + b + c)$

$2c <$ Perimeter

$$c < \frac{\text{Perimeter}}{2}$$

where c = length of the largest side.

Ex. If the perimeter of a triangle is 52 units, then what will be the lengths of all its sides?

(a) <26 (b) <20

(c) <24 (d) <18

Sol. Length of the largest side of the triangle $< \frac{\text{Perimeter}}{2}$

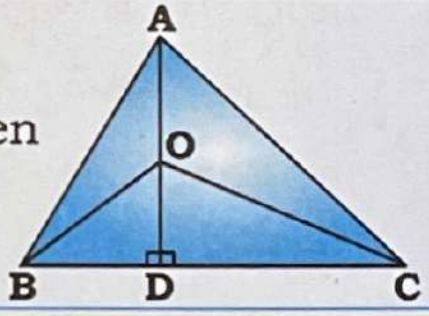
$\Rightarrow C < \frac{52}{2}$

$\Rightarrow C < 26$

In a triangle ABC, AD $\perp$ BC,

AD contains any point O, then

$$\boxed{AB^2 + OC^2 = AC^2 + OB^2}$$



Ex. In a Δ ABC, AD is perpendicular to side BC. On the perpendicular AD, a point O is such that AB = 9 cm, OB = 3 cm, OC = 7 cm. Find the measure of side AC.

Sol. $AB^2 + OC^2 = AC^2 + OB^2$

$9^2 + 7^2 = AC^2 + 3^2$

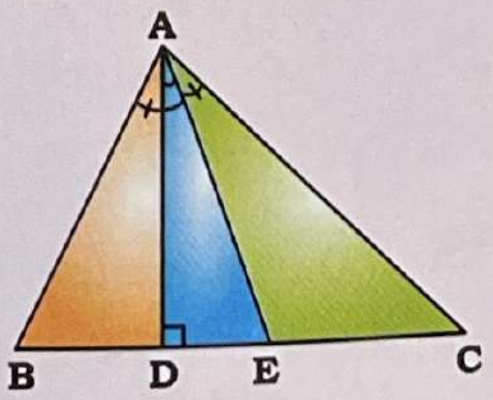
$\therefore$ AC = 11 cm

Based on Angle Bisectors of a Triangle

- In triangle ABC, AE bisects $\angle BAC$ and AD is perpendicular to side BC.

Then

$$\angle DAE = \frac{\angle B - \angle C}{2}$$



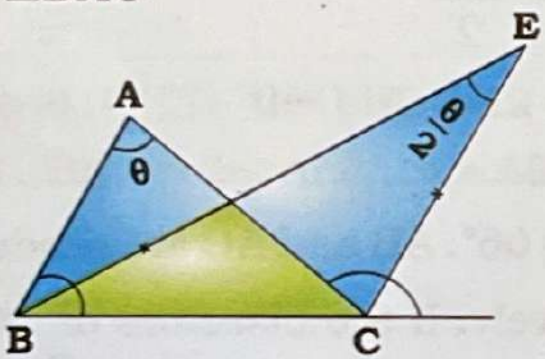
Ex. ΔPQR has $\angle Q = 84°$, $\angle R = 38°$, $PS \perp QR$, and the bisector of $\angle P$ meets at point T on QR. Find the measure of $\angle PTS$.

Sol. $\angle SPT = \dfrac{84° - 38°}{2} = 23°$

$\angle PTS = 180° - 90° - 23° = 67°$

❐ In triangle ABC, if the bisector of interior angle $\angle B$ and the bisector of exterior angle $\angle C$ meet at E then

$$\angle BEC = \frac{1}{2}\angle BAC$$



Ex. ΔPQR has the bisectors of interior angle $\angle Q$ and exterior angle $\angle R$ intersecting at S. If $\angle QSR = 40°$, then $\angle P$ is:

Sol. $\angle QSR = \dfrac{\angle P}{2}$

$40 = \dfrac{\angle P}{2}$

$\boxed{\angle P = 80°}$

❐ In triangle ABC the bisectors of $\angle B$ and $\angle C$ meet at I. Then,

$$\angle BIC = 90° + \frac{\angle A}{2}$$

